PHHQP11 Workshop
Abstract Booklet
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Plenary Talks

Introduction to PT quantum mechanics

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Abstract

In this talk we introduce and review the ideas of PT quantum mechanics at a simple and intuitive level. To illustrate the basic ideas we describe an elementary table-top experiment that we have just completed. This experiment allows us to observe the PT phase transition.

One advantage of PT quantum mechanics is that it allows us to resolve the theoretical problems that have arisen in a number of quantum-mechanical models that have been studied in the past. For example, it solves the unitarity problem in the Lee model and the ghost problem in the Pais-Uhlenbeck and Landau models. We will show that the ideas of PT quantum mechanics can also be used to resolve difficulties that arise in the double-scaling limit of $O(N)$-symmetric quantum theories.

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On some non self-adjoint problems in superconductivity theory

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Abstract

This work was initially motivated by a paper of Yaniv Almog [1]. The main goal is to show how the pseudo-spectrum of some non self-adjoint operators appears in a specific problem occuring in superconductivity [4] and also to prove the non emptyness of the spectrum. These results are obtained together with Y. Almog and X. Pan [2, 3]. These pseudo-spectral methods appear also in the analysis of the Fokker-Planck equation [6]

References

[1] Y. Almog. The stability of the normal state of superconductors in the presence of electric currents. Siam J. Math. Anal. 40 (2), p. 824-850, (2008).

[2] Y. Almog, B. Helffer, and X. Pan. Superconductivity near the normal state under the action of electric currents and induced magnetic field in $\mathbb{R}^2$. *Comm. Math. Phys.*, **300** (1), 147–184 (2010).

[3] Y. Almog, B. Helffer, and X. Pan. Superconductivity near the normal state under the action of electric currents and induced magnetic field in $\mathbb{R}^2_+$. To appear in TAMS.

[4] S. Fournais and B. Helffer. Spectral methods in surface superconductivity. Progress in non-linear PDE, Birkhäuser (2011).

[5] B. Helffer On pseudo-spectral problems related to a time dependent model in superconductivity with electric current. *Confluentes Math.* **3** (2), 237-251, (2011).

[6] B. Helffer and F. Nier. Hypoelliptic estimates and spectral theory for Fokker-Planck operators and Witten Laplacians. Lecture Notes in Mathematics 1862. Springer Verlag (2004).

[7] B. Helffer and J. Sjöstrand. From resolvent bounds to semigroup bounds. Appendix of a paper by Sjöstrand. Proceeding of the Evian Conference in 2009.

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Non-Hermitian operators in quantum physics

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Abstract

We discuss the relevance of non-Hermitian operators in quantum theory in the context of "PT-symmetric quantum mechanics", in which the usual self-adjointness of observables requirement is replaced by their simultaneous Parity-Time invariance. We propose a physical interpretation of a class of Schroedinger operators with non-Hermitian PT-symmetric Robin-type boundary conditions in terms of a (self-adjoint) perfect-transmission scattering problem. Moreover, we establish closed integral-type formulae for similarity transformations relating the non-Hermitian operators with self-adjoint Hamiltonians, succeed in writing down the latter as a simple integro-differential operator and also find a closed formula for the associated "charge conjugation" operator C, which plays the role of fundamental symmetry in a Krein-space reformulation of the problem. Finally, we show that the metric or C operator for the imaginary cubic oscillator and other notorious PT-symmetric operators do not exist.

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Non-Hermitian Maxwell Operators, Lasing, and PT-symmetric scattering

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Abstract

The main focus of this talk will be on the Maxwell operator with gain or loss as a non-hermitian theory, which gives rise to interesting physical phenomena such as laser emission and coherent perfect absorption. We have developed a mathematical framework to describe time-independent lasing states, termed Steady-state Ab initio Laser Theory (SALT) [1]. The lasing state is described by a (possibly non-linear) scattering matrix (S-matrix), which has N poles on the real axis. The lasing modes are eigenstates of this singular S-matrix with eigenvalue tending to infinity, and are calculated conveniently in a biorthogonal basis set of solutions of a non-hermitian linear wave equation, we term this basis set the *constant flux states.* The openness of the laser cavity is treated exactly, and the non-linear mode competition to infinite order; results are in excellent agreement with numerical solutions of the time-dependent non-linear laser equations. Once the S-matrix of SALT is known, quantum fluctuations of the laser fields can be calculated from input-output theory; we will report very recent results deriving a generalization of the Schawlow-Townes-Petermann laser linewidth formula using this approach [2]

Applying the time-reversal operator to the Maxwell operator with gain, maps the lasing problem to a problem of coherent perfect absorption, in which a specific input state of the electromagnetic field (the time-reverse of the lasing mode) is perfectly absorbed in a lossy cavity [3]. This effect has been demonstrated experimentally for a non-trivial example recently [4], and opens up the possibility of coherent control of absorption in complex media. Both lasing and coherent perfect absorption can be combined in systems with PT symmetry (balanced gain and loss). We discuss the properties of PT-symmetric electromagnetic scattering systems, including the appearance of generalized unitarity relations [5] and of spontaneous symmetry breaking of the S-matrix eigenspectrum [6].

References

[1] *Steady-state ab initio laser theory: Generalizations and analytic results*, L. Ge, Y. D. Chong, and A. D. Stone, Physical Review A, **82** 063824 (2010).

[2] *General linewidth formula for steady-state multimode lasing in arbitrary cavities*, Y. D. Chong and A. D. Stone, Physical Review Letters (in press).

[3] *Coherent Perfect Absorbers: Time-reversed Lasers*, Y.D. Chong, L. Ge, H. Cao, and A. D. Stone, Physical Review Letters, **105**, 053901 (2010).

[4] *Time-reversed lasing and interferometric control of absorption*, W. Wan, Y. D. Chong, L. Ge, H. Noh, A. D. Stone and H. Cao, Science, **331**, 889 (2011).

[5] *Conservation relations and anisotropic transmission resonances in one-dimensional PT-symmetric photonic heterostructures*, L. Ge, Y. D. Chong and A. D. Stone, Physical Review A, **85**, 023802 (2012).

[6] *PT-symmetry breaking and laser-absorber modes in optical scattering systems*, Y. D. Chong, L.Ge, and A. D. Stone, Physical Review Letters, **106**, 093902 (2011).

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The quantum Hamiltonian with imaginary cubic potential

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Abstract

The quantum Hamiltonian corresponding to the $x^2 + i\lambda x^3$ potential has been the first PT symmetric, non-Hermitian Hamiltonian whose spectrum has been analyzed. This has led Bessis and Zinn-Justin to conjecture that its spectrum should be real, a conjecture proven later. In this talk, we present a systematic study of the two lowest lying states in the complex λ Riemann surface by a combination of analytic and numerical methods. In the numerical study, a key role is played by the so-called Order Dependent Mapping summation method (introduced by Seznec and Zinn-Justin) applied to the small λ divergent expansion. One particular outcome of the study is a precise determination of about 20 terms of the large λ expansion.

References

[1] U. D. Jentschura and J. Zinn-Justin, J. Math. Phys. **51** (2010) 072106;
ibidem J. Phys. A: Math. Theor. **43** (2010) 425301.

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Specialized Talks

Delay Times and Particle-Like Scattering States in Systems with Loss and Gain

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Abstract

I will speak about our work on the generation of particle-like scattering states in coherent wave-transmission through cavities or disordered media [1]. These waves which we create based on the time-delay operator are highly collimated beams with a trajectory-like wave function and a well-defined delay time. As I will demonstrate in detail, we have recently been able to generalize our approach for generating these states from hermitian to non-hermitian systems. I will discuss the corresponding time-delay operators which we employ in this context and will illustrate the successful operation of our protocol by applying it to systems with uniform absorption as well as to systems with PT-symmetry.

References

[1] S. Rotter, P. Ambichl, and F. Libisch, Phys. Rev. Lett. **106**, 120602 (2011).

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PIP-spaces, metric operators and generalized hermiticity

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Abstract

A metric operator in a Hilbert space $\mathcal{H}$ is a bounded, strictly positive operator G. In that case, G is invertible, G^{-1} is densely defined in $\mathcal{H}$, but not necessarily bounded. If G^{-1} is unbounded, it generates a triplet of Hilbert spaces $\mathcal{H}_{G^{-1}} \subset \mathcal{H} \subset \mathcal{H}_G$, with inner products $\langle G^k \xi \,|\, \eta \rangle$, $k = -1, 0, 1$, respectively (that is, $\mathcal{H}_{G^k}$ is the form domain of G^k), and $\mathcal{H}_{G^{-1}}$ is the conjugate dual of $\mathcal{H}_G$. Then a family of such metric operators (possibly all of them), together with their inverses, generates a lattice of Hilbert spaces, a particular case of partial inner product space (PIP-space) [1]. We shall explore this construction in detail.

This notion of metric operator on $\mathcal{H}$ induces several notions of similarity between operators, G playing the role of an intertwining operator. The most interesting ones among them are those that preserve the spectrum, in particular the discrete spectrum, of an operator that is originally self-adjoint in $\mathcal{H}$. This leads to consider non-self-adjoint operators with real discrete spectrum, an object that has been called a quasi-hermitian or a cryptohermitian operator. Applications of the latter may be found in several non-standard approaches to quantum mechanics, including supersymmetric QM, in the theory of the so-called pseudo-bosons. We expect that PIP-spaces will be a natural framework for these problems.

References

[1] J-P. Antoine and C. Trapani, *Partial Inner Product Spaces - Theory and Applications*; Lecture Notes in Mathematics, vol. 1986, Springer-Verlag, Berlin, Heidelberg, 2009

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Deformations of canonical (anti-)commutation relations, and applications to quantum mechanics

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Abstract

We consider the functional structures connected to suitable deformations of the CCR and of the CAR. We discuss the possible applications of the related *particles* to quatum mechanics

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Coherent states of non-hermitian Hamiltonian

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Abstract

We use the Gazeau-Klauder (G-K) formalism to construct coherent state of a PT-symmetric potential.

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Non-hermitian Hamiltonians and Painlevé IV equation with real parameters

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Abstract

In this talk we will introduce a method to obtain real and complex solutions of the Painlevé IV equation (P_{IV}) with real parameters, based on polynomial Heisenberg algebras and higher-order supersymmetric quantum mechanics. We will study the properties of the resulting solutions, including the analysis of the subspace of the parameter space where non-singular real or complex solutions can be found. In addition, we will analyse the algebras, the eigenfunctions and the spectra of the non-hermitian harmonic oscillator supersymmetric partner Hamiltonians [1,2]. Further investigation on the possible relationship of the generated non-hermitian Hamiltonians with PT-symmetry, pseudo-Hermiticity, and quasi-Hermiticity will also be shown.

References

[1] D. Bermúdez, D.J. Fernández, *Supersymmetric quantum mechanics and Painlevé IV equation*, SIGMA **7** (2011) 025 (14 pages).

[2] D. Bermúdez, D.J. Fernández, *Non-hermitian Hamiltonians and Painlevé IV equation with real parameters*, Phys. Lett. A **375** (2011) 2974-2978.

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Mixed-state dynamics for PT-symmetric quantum systems

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Abstract

The investigation into the time evolution of a PT-symmetric quantum system has thus far been focused primarily into those associated with pure states. However, the state of a quantum system is typically characterised by a mixed-state density matrix. In this talk I will introduce several candidate equations for the dynamics of a density matrix, and discuss their properties. (The talk is based on joint work with E. M. Graefe (Imperial College London).)

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Non-unitary evolution and non hermitian Hamiltonians in decoherence of closed systems

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Abstract

Non-unitary evolutions are essential to explain and study the phenomena of decoherence, the quantum to the classical limit, and the final equilibrium. These phenomena appear in the evolution of quantum system, where decoherence time and relaxation time can be defined using non-unitary evolutions, pole theory, and non-Hermitic Hamiltonians.

As it is indicated in the brief historical summary of paper [6], schematically three periods can be identified in the development of the general program of decoherence:

- In the first period the arrival to the equilibrium of irreversible systems was studied. The main problem of this period was that too long decoherence times were found, if compared with the experimental ones.
- In a second period decoherence in open systems was studied. The main model for this process was EID that determines, case by case, which is the privileged basis, called usually moving preferred basis where decoherence takes place in a decoherence time t_D . This is the orthodox position on the subject [7]. In this period, the decoherence times founded were much smaller, solving the problemof the first period.
- Recently, in a third period, the study of the arrival to equilibrium of closed systems was studied. Within this period, many formalisms for closed systems were introduced (see [8]). Among them a new approach to decoherence, SID, was presented by Castagnino et al., endowed with a non-Hermitian Hamiltonian and therefore with a non unitary evolution. According to this approach decoherence is a process dependent of the choice of some observables which have a particular physical relevance (the van Hove observables) in a closed system. This process also determines which is the privileged basis, called final preferred basis that defines the observables that acquires classic characteristics (at the relaxation time t_R) and that can be interpreted like properties that obey a Boolean logic. SID also solves an important problem explaining how closed system reach equilibrium. Relaxation time can be very long but equilibrium is always reached. In fact, Riemann-Lebesgue theorem shows that, in any system with destructive interference, reaches equilibrium in a very long Khalfin: time [9]. This time was experimentally detected in 2006 [10]. Moreover, in this contribution (following [2]) , we will introduce a generic definition of moving preferred basis where the state decoheres in a time t_D , also solving the main problem of the first period.

In papers [1] we have introduced a proposal of a general definition of moving preferred basis for open systems, using of the usual formalism for these systems known as Environment Induced Decoherence (EID).[1]
In this contribution we will extend our definition to closed system for a particular formalism known as Self Induced Decoherence [2]. If we consider the Hermitic Hamiltonian H of U and the inner product of the evolved state of the proper system and any observable of the proper system we can

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[1] We will call decoherence to the vanishing of the off-diagonal terms in a properly specified basis. We will call relaxation to the decoherence in a final equilibrium basis, i.e. simply equilibrium

make its analytical continuation, in the energy variable into the lower complex half-plane, and in general we will find poles. These poles define all the possible non-unitary decaying modes with characteristic decaying times proportional to the inverse of the imaginary part of the poles (we do not consider the Khalfin mode since it has extremely long decaying time [2]).
From these characteristic times we can deduce the relaxation time, which turns out to be the inverse of the imaginary part of the pole closest to the real axis and therefore it is the largest characteristic time. We can also deduce the decoherence time, that turns out to be a function of the imaginary part of the poles and the initial conditions of the system. Moreover with the same elements a moving preferred basis can be defined, where the state of the system becomes diagonal at the decoherence time [1].
Then, the evolution of the system can be decomposed in three periods.

i. When the time is smaller that decoherence time the closed system has a pure quantum evolution according to a quantum statistic. The non Hermit Hamiltonian corresponding to this non-unitary evolution can be defined.

ii. When the time is greater than the decoherence time and smaller than the relaxation time the state of the closed system becomes diagonal in the moving preferred basis and it evolves according to a classical statistic (then actualization or collapse may occur).

iii. When the time is greater than the relaxation time the system is classical (precisely if collapsed or actualized it evolves according to a classical mechanics) and finally it reaches equilibrium.

We will prove that our definitions coincide with the one of Roland Omnès [3] and we discuss a similar analogy for closed systems: the Casati-Prosen model [4]. Our formalism is essentially based in papers [5].
So in this work we focused the attention on the closed systems approach. Our main aim is to present a new conceptual perspective that will clarify some points that still remain rather obscure in the literature on the subject, e. g. the possibility of decoherence in closed systems and the definition of the moving preferred basis in these systems.

References

[1] M. Castagnino and S. Fortin, On a possible definition of the moving preferred basis, arXiv:1009.0535v2, 2010, submitted to the proceedings of Quantum Physics with Non-Hermitian Operators 2011, Dresden. 2011.

[2] M. Castagnino and R. Laura, Phys. Rev. A, **62**, 022107, 2000; M. Castagnino and O. Lombardi, Phys. Rev. A, **72**, 012102, 2005.

[3] R. Omnès, The Interpretation of Quantum Mechanics, Princeton University Press, Princeton, 1994, R. Omnès, J. Stat. Phys., **53**, 893, 1988, Ann. Phys., **201**, 354, 1990, and Rev. Mod. Phys., **64**, 339, 1992.

[4] G. Casati, T. Prosen, Phys. Rev. A, **72**, 032111, 2005.

[5] M. Castagnino and R. Laura, Phys. Rev. A, **56**, 108, 1997, R. Laura and M. Castagnino, Phys. Rev. A, **57**, 4140, 1998, R. Laura and M. Castagnino, Phys. Rev. E, **57**, 3948, 1998, M. Castagnino, R. Id Betan, R. Laura, R. J. Liotta, J. Phys. A: Math. Gen., **35**, 6055-6074, 2002, M Castagnino et al, Class. Quantum Grav. **25** 154002. 2008.

[6] R. Omnès, Brazilian Journal of Physics, **35**, 207, 2005.

[7] J. Bub, *Interpreting the Quantum World*, Cambridge University Press, Cambridge, 1997.

[8] L. Dioisi, Phys. Rev. Lett. A, **120**, 377, 1987, R. Gambini, J. Pulin, Found. of Phys., **37**, 7, 2007, R. Gambini, R. A. Porto, J. Pulin, Gen. Rel. Grav., **39**, 8, 2007, R. Gambini, J. Pulin, *Modern space-time and undecidability*, in V. Petkov (ed.), *Fundamental Theories of Physics (Minkowski Spacetime: A Hundred Years Later)*, Vol. 165, Springer, Heidelberg, 2010, G. Ford, R. O Connel, Phys. Rev. Lett. A, **286**,87, 2001, D. Polarski and A. A. Starobinsky, Class. Quantum Grav., **13**, 377, 1996, D. Polarski and A.A. Starobinsky, Int.J.Mod.Phys. D, **7**, 455-462, 1998, C. Kiefer and D. Polarski, Ann. Phys. (Leipzig), **7**, 137, 1998, C. Kiefer and D. Polarski, Adv.Sci.Lett., **2**, 164-173,2009.

[9] K. Urbanowski, Eur. Phys. J. D, **54**, 25, 2009, K. Urbanowski, J. Piskorski, arXiv:0908.2219v2, 2009, N. Bleistein, R. Handelsman, *Asymptotic expansion of integrals*, Dover Inc., NewYork, 1986.

[10] C. Rothe, S. I. Hintschich, and A. P. Monkman, Phys. Rev. Lett., **96**, 163601, 2006.

The Variational Perturbation Method applied to a family of anharmonic complex-PT potentials

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Abstract

Several methods of calculation have been elaborated to solve the Schrödinger's equation associated to a given physical system. In the most cases, techniques based on exact calculation proved to be unsuited to solve problems where potentials are complicated. To overcome these difficulties we use generally realistic techniques to approach the studied system. We can cite between them, perturbation methods, variational methods and numerical methods.

Feynman formalism which is an integral formulation of quantum mechanics, have similar difficulties where simple cases have been exactly calculated [1]. New techniques like a coordinates transformation $x \to f(q)$ followed by temporal transformation $t \to s$ should have been developed in order to extend the formalism to less simple cases [2,3].

Besides, in Feynman integral, there is a variational method called Feynman- Kleinert method. It has some formal similarities with Ritz method but differs in the calculations.

Kleinert [5,6] has introduced some improvements in the Feynman method in order to make it more efficient. This technique is based on the matrix density development on the basis of the harmonic oscillator wave functions, and leads to very acceptable results in the case of anharmonical potentials [6].

We present, in the first section, a brief survey on the Feynman-Kleinert variational method. In the second one, we calculate the energy of the ground state relating to a physical system described by a complex potential having a PT-symmetry given by :

$$V(x) = A_2x^2 + iA_1x + iA_3x^3 + A_4x^4$$

where A_1, A_2, A_3 and A_4 are the potential parameters.

We give in the third section, how we can build a complex PT potential from a chosen trial wave function. Finally, our results are compared to those given by the QES method, and some special cases are also discussed.

References

[1] R. P. Feynman and Hibbs, *Quantum Mechanics and Path Integrals* (Mc Graw-Hill, 1965).

[2] A. Diaf, A. Chouchaoui and R. Lombard, Ann. Phys. **317** (2005) 354.

[3] A. Diaf, A. Chouchaoui, Phys. Scr. **84** (2011) 015004.

[4] R. P. Feynman, H. Kleinert, , Phys. Rev. A **34** (1986) 5080.

[5] M. Bachman, H. Kleinert and A. Pelster, Phys. Rev. A **60** (1999) 3429.

[6] H. Kleinert , *Path integrals in Quantum Mechanics, Statistics and Polymer Physics* (World scientific, Singapore, 2009).

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Bound state influence on long-time power law decay in open quantum systems

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Abstract

It is known that quantum systems yield non-exponential (power law) decay on large time scales, associated with continuum threshold effects contributing to the survival probability for a prepared initial state. We study the case in which a bound state of the full Hamiltonian approaches the energy continuum in an open quantum system as the system parameters are varied. We find in this case that at least two regions exist yielding different power law decay behaviors; we term these regions the long time "near zone" and long time "far zone." In the 1-D models we consider, we find that the near zone survival probability falls off according to a t^{-1} power law, and in the far zone as t^{-3}. We show that the time scale T_Q separating these two regions is inversely related to the gap between the discrete bound state energy and the continuum threshold. In the case that the bound state is absorbed into the continuum and vanishes, then the time scale T_Q diverges and the survival probability follows the t^{-1} power law even on asymptotic scales.

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Entangled Quantum State Discrimination using Pseudo-Hermitian System

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Abstract

We demonstrate how to discriminate two non-orthogonal, entangled quantum state which are slightly different from each other by using pseudo-Hermitian system. The positive definite metric operator which makes the pseudo-Hermitian systems fully consistent quantum theory is used for such a state discrimination. We further show that non-orthogonal states can evolve through a suitably constructed pseudo-Hermitian Hamiltonian to orthogonal states. Such evolution ceases at exceptional points of the pseudo-Hermitian system.

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Nonuniqueness of the $\mathcal{C}$ operator in $\mathcal{PT}$-symmetric quantum mechanics

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The $\mathcal{C}$ operator in $\mathcal{PT}$-symmetric quantum mechanics satisfies a system of three simultaneous operator algebraic equations, $\mathcal{C}^2 = 1$, $[\mathcal{C}, \mathcal{PT}] = 0$, and $[\mathcal{C}, H] = 0$ [1, 2]. These equations are very difficult to solve exactly, so perturbative methods have been used in the past to calculate $\mathcal{C}$ [3–6]. The usual approach has been to express the Hamiltonian as $H = H_0 + \epsilon H_1$, and to seek a solution for $\mathcal{C}$ in the form $\mathcal{C} = e^Q\mathcal{P}$, where $Q = Q(q, p)$ is odd in p, even in q, and has the perturbation expansion $Q = \epsilon Q_1 + \epsilon^3 Q_3 + \epsilon^5 Q_5 + \ldots$. In previous work it was always been assumed that the coefficients of even powers of ϵ in this expansion are absent because their presence would violate the condition that $Q(p, q)$ is odd in p [7]. It has recently become clear that the nonuniqueness of the $\mathcal{C}$ operator has important implications for the mathematical and physical interpretation of $\mathcal{PT}$-symmetric quantum mechanics [8]. In an earlier paper [9] it was argued that the $\mathcal{C}$ operator is not unique because the perturbation coefficient Q_1 was nonunique. In this talk, the nonuniqueness of $\mathcal{C}$ is demonstrated at a more fundamental level: It is shown that the perturbation expansion for Q actually has the more general form $Q = Q_0 + \epsilon Q_1 + \epsilon^2 Q_2 + \ldots$ in which *all* powers of ϵ appear and not just odd powers. For the case in which H_0 is the harmonic-oscillator Hamiltonian, Q_0 is calculated exactly and in closed form and is shown explicitly to be nonunique. It is also explained how to calculate the higher coefficients in the perturbation series for Q.

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[1] C. M. Bender, D. C. Brody, and H. F. Jones, Phys. Rev. Lett. **89**, 270401 (2002).
[2] C. M. Bender, Rept. Prog. Phys. **70**, 947-1018 (2007).
[3] C. M. Bender, P. N. Meisinger, and Q. Wang, J. Phys. A: Math. Gen. **36**, 1973 (2003).
[4] C. M. Bender, D. C. Brody, and H. F. Jones, Phys. Rev. D **70**, 025001 (2004).
[5] C. M. Bender, J. Brod, A. Refig, and M. E. Reuter, J. Phys. A: Math. Gen. **37**, 10139-10165 (2004).
[6] C. M. Bender and B. Tan, J. Phys. A: Math. Gen. **39**, 1945 (2006).
[7] H. F. Jones and J. Mateo, Phys. Rev. D **73**, 085002 (2006).
[8] C. M. Bender and S. Kuzhel, J. Phys. A: Math. Theor. (in press).
[9] C. M. Bender and S. P. Klevansky, Phys. Lett. A **373**, 2670 (2009).

Breakdown of adiabatic transfer schemes in the presence of decay or absorption

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Abstract

In atomic physics, adiabatic evolution is often used to achieve a robust and efficient population transfer. Many of the common schemes have also been implemented in optical waveguide structures. Recently there has been increasing interests in the influence of decay and absorption, and their engineering applications. It is often assumed that a small decay or absorption changes the systems behaviour little, albeit introducing an overall decrease of the total norm or the total intensity. However, the effect of decay on the adiabatic behaviour of a quantum system can be drastic. Here it is shown that it can lead to the breakdown of common adiabatic transfer schemes for relatively small decay rates, which only slightly modify the spectrum or the eigenfunctions. In particular, the case of a STIRAP-related scheme that could be easily implemented in optical waveguide structures is investigated. It is found that the transfer property of the scheme breaks down at a sharp threshold, which can be calculated to a good approximation by simple analytical arguments.

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Two $\mathcal{PT}$ symmetry topics

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Abstract

New results are reported on two $\mathcal{PT}$-symmetry topics.

In the first part of the talk, an entanglement related quantum state discrimination scheme is described which is based on the fine-tuned Naimark-dilation of a $\mathcal{PT}$-symmetric subsystem in 2D Hilbert space. (Work together with Carl M. Bender, Dorje C. Brody and Boris F. Samsonov).

In the second part of the talk, the stability properties of nonlinear $\mathcal{PT}$-symmetric 2D plaquettes are discussed. Specific emphasis is laid on the role of the nonlinear terms in shifting the $\mathcal{PT}$-threshold of associated linear systems. This part of the talk is based on common work with Panayotis Kevrekidis, Kai Li and Boris Malomed (arXiv:1204.5530[quant-ph]) as well as on newer results going beyond the findings in this e-print.

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Complex Eigenvalue Problem of the Hamiltonian and the Liouvillian of a Open Quantum Dot System

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Abstract

We discuss complex-eigenvalue problems of the Hamiltonian and the Liouvillian of a T-shaped quantum dot system, an open quantum system. We stress that each eigenstate with a complex eigenvalue breaks the time-reversal symmetry, although the corresponding Schrödinger and Liouville-von Neumann equations observe the time-reversal symmetry. We first show that a complex eigenvalue of the Hamiltonian with a negative imaginary part represents a decaying state, and thereby describes dissipation in quantum-mechanical electronic conduction [1]. The complex-conjugate eigenvalue with a positive imaginary part represents a growing state. The former is called a resonant state and the latter an anti-resonant state. We show that they are defined as eigenstates of the stationary Schrödinger equation under a specific boundary condition.

We next show that the same argument applies to the complex-eigenvalue problem of the Liouvillian, which now can have mixed-state solutions [2]. The mixed eigenstate with a negative imaginary part may describe the approach to equilibrium. We map the problem to the Schrödinger equation of two distinguishable particles, namely the ket particle and the bra particle. The mapping introduces an interaction between the two particles, which can lead us to eigenvalues that cannot be deduced from the pure-state solutions of the original Schrödinger equation.

Specifically, we consider the open quantum system given by the Hamiltonian

$$H = -t \sum_{x=-\infty}^{+\infty} \left(c_{x+1}^\dagger c_x + c_x^\dagger c_{x+1} \right) - t_1 \left(c_0^\dagger c_d + c_d^\dagger c_0 \right),$$

where $c_x^\dagger$ and c_x are the creation and annihilation operators of a particle on the site x on the infinite lead, whereas $c_d^\dagger$ and c_d are those on the dot site "d." The Schrödinger equation with this Hamiltonian yields complex eigenvalues that correspond to resonant and anti-resonant states.

The resonant state of the Hamiltonian spontaneously breaks the time-reversal symmetry and supports the arrow of time. In order to describe relaxation to the equilibrium, however, we must work with the Liouville-von Neumann equation

$$i\hbar \frac{d\rho}{dt} = [H, \rho] \stackrel{\text{def}}{=} L\rho$$

because of the equilibrium is not a state in the Schrödinger framework but one in the Liouville framework, namely the density operator ρ. We have indeed obtained a non-trivial complex eigenvalue, which may describe the decay of a two-level atom in a resonating laser. The point here is that we do not introduce any approximations thanks to the fact that our model of the open quantum system is simple enough to treat numerically exactly.

References

[1] N. Hatano, K. Sasada, H. Nakamura and T. Petrosky, Prog. Theor. Phys. **119** (2008) 187-222.

[2] R. Nakano, N. Hatano and T. Petrosky, Int. J. Theor. Phys. **50** (2011) 1134-1142.

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The Physics of exceptional points

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Abstract

After a short rehash about the nature of exceptional points their ubiquitous occurrence in a great variety of physical problems is presented. A few partic- ular cases such as their first identification in the laboratory and their role in quantum phase transitions and quantum chaos are presented in more detail.

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Universal behaviour in higher derivative theories with $x^2(ix)^\epsilon$ potentials

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Abstract

The PT-symmetric Hamiltonian $H = p^2 + x^2(ix)^\epsilon$ (ϵ real) exhibits a phase transition at $\epsilon = 0$. When $\epsilon \geq 0$, the eigenvalues are all real, positive, discrete, and grow as ϵ increases. However, when $\epsilon < 0$ there are only a finite number of real eigenvalues. As ϵ approaches -1 from above, the number of real eigenvalues decreases to one, and this eigenvalue becomes infinite at $\epsilon = -1$. In this talk we explore these qualitative spectral behaviours and show that they are generic and that they are exhibited by the eigenvalues of the general class of Hamiltonians $H^{(2n)} = p^{2n} + x^2(ix)^\varepsilon$ (ϵ real, $n = 1, 2, 3, \ldots$). The complex classical behaviours of these Hamiltonians will also be explained.

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Universal chirality in PT symmetric rings and lattices

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Abstract

Over the past decade, PT symmetry breaking and it's consequences have been extensively explored in tight-binding lattices as well as continuum models. Most of this work has focused on time evolution of the system across the PT symmetric phase boundary and the attendant violation of unitarity which leads to exponentially increasing net intensity in the PT broken phase. I will discuss a new observable, the average momentum or chirality, and it's evolution across the PT phase boundary in PT lattices. I will show that the momentum acquires a maximum universal value at the PT transition point, and discuss its consequences.

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WKB for PT-symmetric Sturm-Liouville Problems

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Abstract

It is shown that the WKB method, with the WKB path passing through either one turning point or a pair of turning points, is capable of reproducing with great accuracy, even for small coupling, the spectra of various non-Hermitian PT-symmetric Hamiltonians on a finite interval with Dirichlet boundary conditions. The interesting structure, including the exceptional points where a pair of real eigenvalues coalesce and become a complex-conjugate pair, arises in these WKB approximations through an interplay between the two terms in the approximate secular equation.

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Deriving Non-Hermitian Quantum Theory from Covariance and the Correspondence Principle

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Abstract

As a starting point it is demonstrated on the basis of the non-Hermitian Klein-Gordon equation, how covariance induces a pair of time-dependent Schrödinger equations. These so-called retarded and advanced Schrödinger equations reduce for Hermitian Hamilton operators to the well known time-dependent Schrödinger equation and its Hermitian conjugate. It is also shown via the separation ansatz that the retarded and advanced Schrödinger equation share even for non-Hermitian Hamilton operators the same stationary Schrödinger equation. Time-dependent wave functions solving the retarded or advanced Schrödinger equation can be expanded in terms of a suitable set of eigenfunctions of the stationary Schrödinger equation being orthogonal under integration along some complex contour interconnecting two suitable anti-Stokes cones induced by the long-distance part of the interaction potential. For matrix Hamiltonians this orthonormal set of stationary eigenfunctions reduces to the right and left eigenbasis of the respective matrix Hamiltonians. A suitable combination of the time- dependent retarded and advanced Schrödinger equation leads to some continuity equation yielding in correspondence to classical mechanics in the complex spacial plane some probabilistic interpretation of our non-Hermation Quantum Theory. Throughout the presentation the formalism being easily generalized also to Quantum Field Theory is demonstrated and tested by applying it to a list of suitable problems.

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Semi-classical physics and diffraction in micro-Lego-lasers

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Abstract

Summary: *Organic micro-lasers are model systems for addressing scattering theory and wave-quantum chaos physics. We fabricate and characterize optically highly open resonators which shape (including surface quality and singularity points) is critical for lasing dynamics. Both semi-classical and diffraction behaviours are observed with our experimental method.*

Dielectric resonators are widely used in photonics research [1]. We fabricate active resonators, inserting laser dye molecules within the cavity. Various shapes of resonators are studied and proved to have a prominent effect on the optical lasing states [2,3]. From a photonic point of view, a resonator is a cavity or an open billiard which inside region can be described as a potential well of finite depth. Quantum chaos techniques are then particularly appropriate to describe these dynamical systems [4] so that our devices appear as test-bed systems for semi-classical theory and especially trace formulas.

Indeed both the presence of the laser dye causing a homogeneous gain and the cavity boundary acting as mirrors and sources of diffraction contribute to select some trajectories along which the light is amplified, giving birth to laser light. Actually these periodic orbits emerge in crucial features of wave systems [2] and in our case they are enlightened in the detected spectra and far field patterns. This semi-classical approach is valid since we consider cavity sizes of the order of 100 wavelengths.

The micro-lasers are designed by electron-beam lithography, which provides etching quality at the nanometer scale. The emission is collected in the plane of the cavity. Additional information is collected through the influence of the pump polarization and laser thresholds.

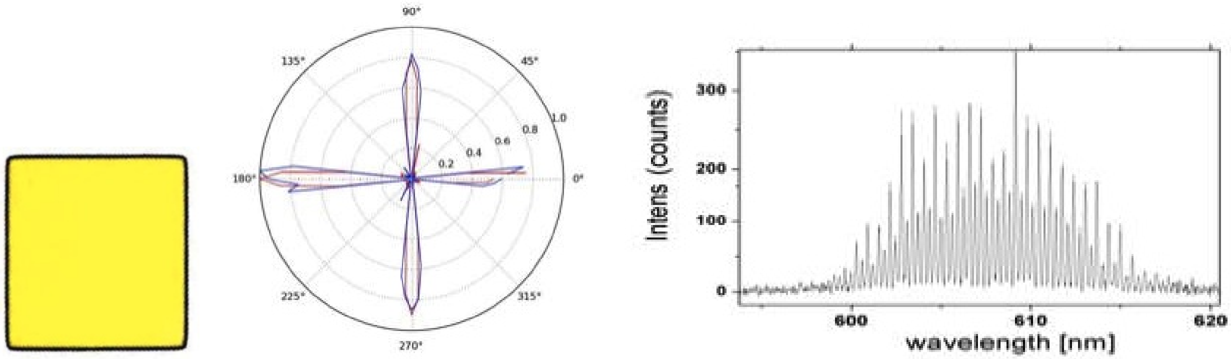

Figure 1: Left, photography from an optical microscope of a planar square resonator. Center, far-field emission diagram of a square resonator. The lobes are parallel to the square sides. Right, experimental emission spectrum from a triangle microlaser.

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References

[1] *Practical applications of microresonators in optics and photonics*, ed. A. Matsko (CRC Press, Boca Raton, 2009).

[2] M. Lebental, N. Djellali, C. Arnaud, J.-S. Lauret, J. Zyss, R. Dubertrand, C. Schmit, and E. Bogomolny, Phys. Rev. A, **76**, 023830, 2007.

[3] E. Bogomolny, N.Djellali, R. Dubertrand, I. Gozhyk, M.Lebental, C. Schmit, C. Ulysse and J. Zyss, Phys. Rev. E, **83**, 036208 (2011).

[4] H.-J. Stöckmann, *Quantum chaos, an introduction* (Cambridge university press, 1999).

Solvable $\mathcal{PT}$-symmetric potentials beyond the shape-invariant class

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Abstract

The most well-known solvable potentials of quantum mechanics (harmonic oscillator, Coulomb, Morse, Pöschl–Teller, etc.) belong to the so-called shape-invariant class. The $\mathcal{PT}$-symmetric version of these potentials has been constructed in the early years of $\mathcal{PT}$-symmetric quantum mechanics, both with real [1] and complex [2] energy spectrum. Further aspects of $\mathcal{PT}$-symmetric quantum mechanics have also been studied for them, such as the pseudo-norm, quasi-parity, $\mathcal{C}$ operator and the spontaneous breakdown of $\mathcal{PT}$ symmetry. This latter topic led to the finding that the spontaneous breakdown of $\mathcal{PT}$ symmetry occurs in these systems in a sudden way, i.e. all the energy eigenvalues of a specific potential turn into complex at the same parameter value. This is markedly different from numerically, or semi-analytically solvable potentials, e.g. the Bender–Boettcher-type potentials and the $\mathcal{PT}$-symmetric square well.

Inspired by these studies recently we constructed a four-parameter Natanzon-class potential that is solved by the Jacobi polynomials [3] and which contains all the three-parameter shape-invariant potentials (Scarf I, II, Rosen–Morse I, II, Eckart, generalized Pöschl–Teller). Two of the parameters, C and δ control a $z(x)$ function that transforms the appropriate Schrödinger equation into the Jacobi differential equation. Shape-invariant potentials are obtained in the limits $\delta \to 0$ and $|\delta| \to \infty$, $C/\delta = const.$ The (C, δ) parameter space can be divided into four quadrants depending on the sign of C and δ. In [3] we discussed the $C < 0$, $\delta \geq 0$ case, which contains the Scarf II and Rosen–Morse I limits. We demonstrated that in the $\mathcal{PT}$-symmetric setting the spontaneous breakdown of $\mathcal{PT}$ symmetry occurs gradually, starting from the lowest state.

Here we discuss the remaining three combinations and find that they all demonstrate specific aspects of $\mathcal{PT}$ symmetry. For $C < 0$, $\delta \leq 0$ the problem has to be defined on the imaginary shifted x axis $x - \mathrm{i}c$, because the potentials (including the Eckart and generalized Pöschl–Teller limits) are singular at the origin. For $C > 0$, $\delta \leq 0$ normalizability requires that the problem is defined on specific trajectories in the complex x plane, similarly to some Bender–Boettcher-type potentials. Furthermore, for $\delta = -1$ the $\mathcal{PT}$-symmetric Dutt–Khare–Varshni potential [4] is recovered as a further special limit. These examples indicate that earlier results on specific $\mathcal{PT}$-symmetric potentials can be elegantly unified within the framework of Natanzon-class potentials.

References

[1] G. Lévai, M. Znojil, J. Phys. A **33** (2000) 7165.

[2] G. Lévai, M. Znojil, Mod. Phys. Lett. A **30** (2001) 1973.

[3] G. Lévai, J. Phys. A, *in press.*

[4] M. Znojil, G. Lévai, P. Roy, R. Roychoudhury, Phys. Lett. A **290** (2001) 249.

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Energy-dependent potentials

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Abstract

Two properties of energy dependent potentials will be discussed. The first one concerns the regularization effect of the energy dependence on singular potentials. The second is devoted to supersymmetry.

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Multimoded $\mathcal{PT}$-symmetric optical structures

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Abstract

We theoretically investigate the unusual characteristics $\mathcal{PT}$-symmetric optical potentials that support many guided modes. The existence of several spatial supermodes leads to the formation of $\mathcal{PT}$-islands in the eigenvalue parametric space. Multiple $\mathcal{PT}$-thresholds, complex bifurcation diagrams, vortices in the transverse Poynting power flow around phase singularities, are few of the peculiar features of multimode $\mathcal{PT}$-Optics. Propagation dynamics aspects are also examined.

References

[1] K. G. Makris, R. El-Ganainy, D. N. Christodoulides and Z. H. Musslimani, *Beam dynamics in $\mathcal{PT}$ symmetric optical lattices*, Phys. Rev. Lett. **100**, 103904 (2008).

[2] K. G. Makris, R. El-Ganainy, D. N. Christodoulides and Z. H. Musslimani, *$\mathcal{PT}$ symmetric optical lattices*, Phys. Rev. A **81**, 063807 (2010).

[3] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev and D. Kip, *Observation of paritytime symmetry in Optics*, Nature Physics **6**, 192 (2010).

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Complex Classical Mechanics of a QES System

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Abstract

We consider the PT invariant non-Hermitian potential $V(x) = -(\xi \cosh 2x - iM)^2$ in the framework of complex classical mechanics. The quantum mechanical system corresponding to this potential is quasi-exactly solvable (QES) for positive integer values of M and exhibits PT symmetry breaking (unbreaking) spontaneously for even (odd) values of M. We study this potential in a complex plane to demonstrate different quantum effects. For real energy the particle makes closed orbits around one of the wells depend- ing on the initial condition. However surprisingly the particle can have open orbits even with real energy if it is initially placed in certain region between the two wells in the same side of the imaginary axis. On the other hand when the particle energy is complex the trajectory is open and the particle tunnel back and forth between two wells which are separated by classically forbidden path. At the classical level unlike the analogous quantum situation we do not see any qualitative differences in the features of the particle dynamics for M-even (PT symmetry broken phase) and M-odd (PT unbroken phase). The tunneling time is calculated for different pair of wells and is shown to vary inversely with the imaginary component of energy.

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PT symmetry as a necessary and sufficient condition for unitary time evolution

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Abstract

While Hermiticity of a time-independent Hamiltonian leads to unitary time evolution, in and of itself, the requirement of Hermiticity is only sufficient for unitary time evolution. In this paper we provide conditions that are both necessary and sufficient. Specifically, we show that PT symmetry of a time-independent Hamiltonian, or equivalently, reality of the secular equation that determines its eigenvalues, is both necessary and sufficient for unitary time evolution. For any PT-symmetric Hamiltonian H there always exists an operator V that relates H to its Hermitian adjoint according to $VHV^{-1} = H^{\dagger}$. When the energy spectrum of H is complete, Hilbert space norms $\langle \psi_1 | V\psi_2 \rangle$ constructed with this V are always preserved in time. With the energy eigenvalues of a real secular equation being either real or appearing in complex conjugate pairs, we thus establish the unitarity of time evolution in both cases. We also establish the unitarity of time evolution for Hamiltonians whose energy spectra are not complete. We show that when the energy eigenvalues of a Hamiltonian are real and complete the operator V is a positive Hermitian operator, which has an associated square root operator that can be used to bring the Hamiltonian to a Hermitian form. In addition, we show that systems with PT-symmetric Hamiltonians obey causality. We note that Hermitian theories are ordinarily associated with a path integral quantization prescription in which the path integral measure is real, while in contrast non-Hermitian but PT-symmetric theories are ordinarily associated with path integrals in which the measure needs to be complex, but in which the Euclidean time continuation of the path integral is nonetheless real. We show that PT symmetry generalizes to higher-derivative field theories the Pauli-Weisskopf second-quantized prescription for stabilizing the second-order Klein-Gordon theory against transitions to states with negative frequency.

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Quasi-Hermitian Hamiltonians associated with exceptional orthogonal polynomials

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Abstract

Recently, there has been renewed interest in the analysis of solvable Hamiltonians in quantum mechanics due to the discovery of Exceptional Orthogonal Polynomials. Exceptional X_m $(m \geq 1)$ orthogonal polynomials are the infinite sequence of eigenfunctions of a certain class of Sturm-Liouville problems with rational coefficients. Unlike the classical orthogonal polynomials, these new polynomials have the remarkable properties that they still form complete sets with respect to some positive definite measure, although they start with degree $n \geq m$.

In this communication, our aim is to generate some exactly solvable Hamiltonians which are non-Hermitian in nature and whose bound state wave functions are associated with Laguerre and Jacobi-type exceptional orthogonal polynomials. These Hamiltonians are shown, with the help of imaginary shift of coordinate: $e^{-\alpha p} x e^{\alpha p} = x + i\alpha$, to be both quasi and pseudo-Hermitian. It turns out that the corresponding energy spectra is entirely real.

Keywords: Exceptional Orthogonal Polynomial, quasi-Hermitian, Pseudo-Hermitian

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Pseudo-Hermitian Quantum Systems Defined by an Unbounded Metric Operator

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Abstract

In this talk I give a general overview of Pseudo-Hermitian Quantum Mechanics and present a complete solution for the notorious problem of formulating pseudo-Hermitian quantum systems using an unbounded metric operator. This is one of the rare examples of a problem of basic importance that was discussed in various occasions (including the first of this series of meetings, i.e., PHHQP1, in 2003), but resisted a satisfactory resolution until recently.

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The Pauli equation with complex boundary conditions

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Abstract

We consider one-dimensional Pauli Hamiltonians in a bounded interval with possibly non-self-adjoint Robin-type boundary conditions. We study the influence of the spin-magnetic interaction on the interplay between the type of boundary conditions and the spectrum. A special attention is paid to $\mathcal{PT}$-symmetric boundary conditions with the physical choice of the time-reversal operator $\mathcal{T}$.

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Pseudo Hermitian interactions in the Dirac equation

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Abstract

It is shown that complex vector potentials (leading to complex magnetic fields) can produce bound states in the mass-less Dirac equation in $(2+1)$ dimensions. In particular, it is shown that such magnetic fields lead to real eigenvalues and the Dirac Hamiltonian(s) are η-pseudo Hermitian. A couple of examples have been considered.

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On the origin of nuclear clustering

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Abstract

Nuclear clustering is arguably one of the most mysterious nuclear phenomena. The challenging problem is how to separate generic and specific features of this intricate many-body phenomenon. The energetic order of particle emission thresholds, and their nature, is given by the neutron- and proton-dependence of the nuclear binding energy, i.e., it depends on specific properties of the nuclear Hamiltonian. The appearance of specific decay channels involving both kinds of nucleons, and the absence of stable clusters/nuclei entirely composed of like nucleons, is a direct consequence of the isospin structure of the nuclear force. The continuum coupling effects play a rather minor role in the interplay between various contributions to the total binding energy, at least close to the valley of stability. On the other hand, the phenomenological rule that cluster correlations are seen only in the vicinity of the respective cluster emission threshold is unlikely a consequence of specific properties of nuclear forces and calls for a generic explanation.

We claim that the nuclear clustering is an emergent, near-threshold phenomenon, which involves interaction of Shell Model (SM) states via the decay channel(s) and cannot be elucidated within the Closed Quantum System (CQS) framework. This anti-Hermitian coupling leads to the formation of the aligned state, the eigenstate of the Open Quantum System (OQS) which captures most of the continuum coupling, and, above the decay threshold, exhausts most of the decay width. In realistic nuclear applications, this interaction may lead to collective phenomena in the ensemble of SM states. The collectivity of an aligned state is a fingerprint of instability in an ensemble of all SM states having the same quantum numbers and coupled to the same decay channel.

For large angular momenta l and/or charged particle decays, the energy window for the formation of a collective aligned state is pushed by centrifugal and Coulomb barriers above the decay threshold. We will demonstrate that the point of maximum continuum coupling for the charge-particle decay is related to the turning point of all Exceptional Threads (ETs), i.e., the trajectories of EPs, involving the aligned state. The energy of this turning point is independent of the continuum-coupling strength but it depends both on the potential radius and the height of the Coulomb barrier. This means that one can predict the position of the energy interval of maximum continuum coupling for any charge-particle decay channel.

Very narrow aligned α-decaying states can be found around the turning point in quite a few p- and sd- shell nuclei. The Hoyle resonance in ^{12}C represents a splendid example of such a continuum-correlated collective state.

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Experimental PT-symmetry breaking in a large-scale optical fiber network

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Abstract

We experimentally investigate light transport in an extended network of coupled optical fibers which is a discrete-time analog of photonic waveguide arrays. The implementation of this concept in the temporal domain allows us to realize appropriate gain and loss distributions and to superimpose an optical potential as demanded by parity-time (PT) symmetry. In this system, the transition from exact to broken PT phases is explored in a two-dimensional parameter scan. We observe PT Bloch oscillations and discuss the possibility of superluminal energy transport. Moreover, we perform scattering experiments to demonstrate the unidirectional invisibility of PT Bragg elements.

Optical structures have been proposed [1] as an ideal platforms to realize PT-symmetric [2] dynamics in a fully classical system by implementing a complex optical potential $n(x) = n^*(-x)$. This correspondence between optics and quantum theory is enabled by an isomorphism of the involved wave equations. So far, optical PT symmetry has only been reported for elemental two-component systems [3, 4], whereas large- scale PT lattices were hindered by technical difficulties. We overcome these obstacles by using a setup of coupled fiber loops which transfers the concept of optical lattices to the temporal domain [5,6]. Our system evolves in discrete steps and is fully governed by basic iteration equations. The required distributions of gain and loss and of the phase potential can be readily implemented via temporal modulation. This has allowed the first demonstration of optical dynamics in large-scale PT-synthetic optical lattices [7]. Apart from the well-known PT phase transition, we observed PT Bloch oscillations [8] and unidirectional invisibility [9].

We acknowledge financial support from DFG Forschergruppe 760, the Cluster of Excellence Engineering of Advanced Materials, SAOT and the German-Israeli Foundation. This work was also supported by NSF ECCS-1128520 and by AFOSR grant FA95501210148.

References

[1] Makris, K.G., El-Ganainy, R., Christodoulides, D.N. & Musslimani, Z.H. *Beam Dynamics in PT Symmetric Optical Lattices.* Phys. Rev. Lett. **100**, 103904 (2008).

[2] Bender, C.M. & Boettcher, S. *Real Spectra in Non-Hermitian Hamiltonians Having PT Symmetry.* Phys. Rev. Lett. **80**, 5243-5246 (1998).

[3] Rüter, C.E., Makris, K.G., El-Ganainy, R., Christodoulides, D.N., Segev, M. & Kip, D. *Observation of paritytime symmetry in optics.* Nature Physics **6**, 192-195 (2010).

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[4] Guo, A. et al. *Observation of PT-Symmetry Breaking in Complex Optical Potentials.* Phys. Rev. Lett. **103**, 093902 (2009).

[5] Regensburger, A., Bersch, C., Hinrichs, B., Onishchukov, G., Schreiber, A., Silberhorn, C. & Peschel, U. *Photon Propagation in a Discrete Fiber Network: An Interplay of Coherence and Losses.* Phys. Rev. Lett. **107**, 233902 (2011).

[6] Schreiber, A., Cassemiro, K.N., Potocek, V., Gbris,A., Mosley, P.J., Andersson, E., Jex, I. & Silberhorn, C. *Photons Walking the Line: A Quantum Walk with Adjustable Coin Operations.* Phys. Rev. Lett. **104**, 050502 (2010).

[7] Regensburger, A., Bersch, C., Miri, M. A., Onishchukov, G., Christodoulides, D.N. & Peschel, U. *PT-Synthetic Photonic Lattices.* Nature (to be published) doi:10.1038/nature11298.

[8] Longhi, S. *Bloch Oscillations in Complex Crystals with PT Symmetry.* Phys. Rev. Lett. **103**, 123601 (2009).

[9] Lin, Z., Ramezani, H., Eichelkraut, T., Kottos, T., Cao, H. & Christodoulides, D.N. *Unidirectional Invisibility Induced by PT-Symmetric Periodic Structures.* Phys. Rev. Lett. **106**, 213901 (2011).

Landau-Zener-Stückelberg interferometry in $\mathcal{PT}$-symmetric optical waveguides

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Abstract

We investigate theoretically the non-adiabatic transitions in a $\mathcal{PT}$-symmetric lattice subject to a strong ac force . In an optical realization of the lattice the curvature of the waveguides along the paraxial propagation direction plays the role of the external periodic force necessary to drive the system through an avoided level crossing. Analytical expressions for the transition probabilities after multiple passages are obtained within an effective two-mode approximation. We show that gain and losses of the light beam, as well as the relative occupation probabilities of the bands involved in the transitions can be accurately managed upon tuning the parameters of the system and are particularly sensitive to the amplitude of the non-Hermitian component of the lattice. Numerical simulations for the complete system are found to agree very well with the approximate analytical results.

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Perturbative Analysis of Spectral Singularities

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Abstract

Spectral singularities are well-known mathematical concept with interesting physical counterpart; it corresponds to a zero-width resonance. Such resonances can be realized in optical systems consisting of a gain medium. They give rise to a particular lasing effect that occurs at the gain threshold. In particular, it turns out that a time-reversed copy of an optical spectral singularity (OSS) corresponds to a coherent perfect absorption of light. This is the basic physical phenomenon occurring in an antilaser. We develop a perturbative method of computing spectral singularities of a Schrodinger operator defined by a general complex potential that vanishes outside a closed interval. These can be realized as zero- width resonances in optical gain media and correspond to a lasing effect that occurs at the threshold gain. Their time-reversed copies yield coherent perfect absorption of light that is also known as an antilaser. We use our general results to establish the exactness of the n-th order perturbation theory for an arbitrary complex potential consisting of n deltafunctions, obtain an exact expression for the transfer matrix of these potentials, and examine spec- tral singularities of complex barrier potentials of arbitrary shape. In the context of optical spectral singularities, these correspond to inhomogeneous gain media.

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Exceptional points in open and closed gain-loss structures

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Abstract

I will speak about our recent progress in understanding the relation between exceptional points in closed and open gain-loss structures, respectively. In particular, I will focus on exceptional points in the spectrum of a closed-system Hamiltonian and relate them to exceptional points in the spectrum of the scattering matrix for the corresponding open system. Our results show in detail how these seemingly different situations share a very close and elegant connection with each other. In the second part of my talk I will speak about pump-induced exceptional points which can strongly influence a laser's above-threshold characteristics. In the vicinity of these exceptional points a laser can turn off even when the applied pump power is increased. Such counter-intuitive signatures of pump-induced exceptional points can be experimentally probed with coupled ridge or microdisk lasers.

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Effects of nonuniform strain on $\mathcal{PT}$-symmetric photonic crystals

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Abstract

I describe the effects of $\mathcal{PT}$-symmetry breaking nonuniform strain on a photonic crystal with balanced amplification and absorption. The spectrum of this system is governed by the combination of supersymmetry and a hidden generalized time-reversal symmetry of a dynamical origin, which applies to all states but the ground state.

Joined work with Nicole Yunger Halpern, Lancaster.

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Non-hermitian propagation of coherent states

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Abstract

We study the time evolution of coherent states for non-Hermitian Hamiltonians. We find that the semiclassical limit leads to an underlying classical dynamics on (real) phase space which is no longer Hamiltonian. Instead it is governed by a combination of a Hamiltonian vectorfield and a gradient vectorfield which are coupled by a time dependent metric. We discuss this dynamics and compare it with complex Hamiltonian dynamics on a complexified phase space. We will concentrate in particular on the quadratic case and show how the above dynamics is related to a projection of the complexified Hamiltonian dynamics and a special complex structure on phase space. This is joint work with Eva-Maria Graefe.

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Spectral and Nodal Properties of a Quasi-Exactly Solvable Sextic Anharmonic Oscillator

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Abstract

The properties of the quantum problem with potential $V(x) = cx^2 + x^6$ are studied. This problem is insolvable except at discrete negative integer values of c where a solvable multiplet exists and the number of eigenfunction nodes is finite. At the insolvable coupling $c = 0$, Hille has shown that the number of nodes is infinite with almost all in the complex spacial domain. Based on the assumption that the number of nodes is infinite at all insolvable couplings, the singular behavior is studied that is necessary to remove and then to reinstate infinite numbers of nodes when c passes through a solvable value.

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Linear and Non-linear Supersymmetry for Non-Hermitian Matrix Hamiltonians

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Abstract

We study intertwining relations for matrix non-Hermitian Hamiltonians by matrix differential first order and higher order operators. We find conditions under which the product of mutually conjugate (in some sense) intertwining operators is a polynomial of the Hamiltonian. Factorization of a matrix differential intertwining operator into product of matrix differential first order intertwining operators with singular, in general, coefficients is proved. The problem of minimization of an matrix differential intertwining operator is considered and criterion of minimizability is presented. Appearance of matrix hidden symmetry operator is investigated.

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From Kreĭn to S–spaces: a short step

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Abstract

S-spaces generalize commonly known Kreĭn spaces. If Kreĭn spaces can be thought of as *real* ones (the operator J is a real unitary), S-spaces are in fact their *complexifications.* Some properties can be extended to S-spaces while some others provide with new phenomena.

References

[1] FH Szafraniec, Two-sided weighted shifts are 'almost Krein' normal *Oper. Theory Adv. Appl.* **188** (2008), 245–250.

[2] Friedrich Philipp, FH Szafraniec and Carsten Trunk, Selfadjoint operators in S-spaces *J. Functional Analysis* **260** (2011) 1045–1059.

[3] FH Szafraniec, *Dissymmetrising*[1] inner product spaces, in progress.

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[1] The neologism 'to dissymmetrize' is something which fits in with our intention; fortunately English is flexible enough to let this kind of creation to happen.

Exceptional points behind eigenspace and eigenvalue anholonomies of bound states

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Abstract

An adiabatic cycle of quantum system can induce discrepancies. A famous example is the Berry phase, also known as the phase anholonomy [1]. In Ref. [2], it is shown that exotic anholonomies appear in a family of closed systems: When we keep track of a bound state adiabatically along a cycle, the initial and final eigenspaces can be different, even when spectral degeneracies are absent. Furthermore, such an anholonomy in the eigenspaces induces another anholonomy in the eigenenergies. Recently, examples of the systems that exhibit the eigenspace and eigenvalue anholonomies has been found in various systems including periodically driven systems and quantum circuits [3]. Also, a gauge theoretical formulation to unify the phase and eigenspace anholonomies has been introduced [4].

What is the origin of the adiabatic quantum anholonomies? As for the Berry phase, it has been known that an effective magnetic monopole located in a degeneracy points play a crucial role [1]. In contrast, the minimal example of the exotic anholonomies has no degeneracy point in the parameter space [5]. We will explain that the *complexification* of the parameter is the key to answer this question: exceptional points in the complexified parameter space governs the eigenspace and eigenvalue anholonomies [6].

This is a joint work with Sang Wook Kim (Pusan National University) and Taksu Cheon (Kochi University of Technology).

References

[1] M. V. Berry, Proc. R. Soc. London **A** **392** 45 (1984).

[2] T. Cheon, Phys. Lett. A **248** 285 (1998).

[3] AT, S. W. Kim and T. Cheon, EPL **96** 10005 (2011) and references therein

[4] T. Cheon and AT, EPL **85** 20001 (2009); AT, T. Cheon and S. W. Kim, arXiv/1203.5412 (2012).

[5] AT and M. Miyamoto, Phys. Rev. Lett. **98** 160407 (2007).

[6] S. W. Kim, T. Cheon and AT, Phys. Lett. A **374** 1958 (2010).

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On the spectrum of a magnetic chain graph

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Abstract

We study Schrödinger operators on an infinite quantum graph of a chain made of identical rings connected at the touching points by δ-couplings with a parameter $\alpha \in \mathbb{R}$. The graph is placed into a magnetic field perpendicular to the loop plane. We consider a "bending" deformation of the chain consisting of changing one position at a single ring and show that it gives rise to eigenvalues in the open spectral gaps. We analyze dependence of these eigenvalues on the coupling α and the "bending angle" as well as resonances of the system coming from the bending.

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Weak and generalized Weyl form of the commutation relations for unbounded operators

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Abstract

The commutation relation $[S, T] = I$ for two unbounded operators can be formulated in several nonequivalent ways. The *weak* sense is given in terms of the inner product where the operators act. In the case where $T \neq S^*$, two generalized *number operators* can be defined in natural way and their point spectra can be easily described.

On the other hand, the generalized Weyl form of the commutation relation requires (as in the classical case where self-adjoint operators are considered) more regularity of the involved operators, since they must be, first of all, the generators of semigroups of bounded operators. We will show that this form of the commutation relation introduces severe contraints to the spectra of the operators S, T.

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On the analytical solvability of (polynomial) 1D Schrödinger equations

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Abstract

We review the exact solvability of stationary 1D Schrödinger equations, for polynomial potentials. An exact form of complex-WKB method, assisted by zeta-regularisation, extracts a set of exact quantization conditions directly from Wronskian identities, like the Bethe Ansatz for integrable systems. Such quantization formulae do not only fix the eigenvalues; some evaluate the spectral determinants, and others the wavefunctions (all exactly), at any position of the spectral parameter.

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Time-dependent $\mathcal{PT}$-symmetric quantum mechanics

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Abstract

In conventional quantum mechanics, the time evolution is governed by the Schrödinger equation. In $\mathcal{PT}$-symmetric quantum mechanics, the inner-product is dynamically determined by the Hamiltonian. When the Hamiltonian is time-dependent, one should expect a time-dependent metric operator. In this case, it can be shown that the time-evolution must be modified. In this talk, we will first propose a criterion to determine the time-evolution equation in time-dependent $\mathcal{PT}$-symmetric quantum mechanics. Then we will construct a map between conventional and $\mathcal{PT}$-symmetric quantum systems. Last we will apply these ideas to various time-dependent phenomena.

References

[1] J. Gong and Q.-h. Wang, "Geometric Phase in $\mathcal{PT}$-Symmetric Quantum Mechanics" *Phys. Rev. A* **82**, 012103 (2010) [arXiv:1003.3076]

[2] J. Gong and Q.-h. Wang, "Time-dependent $\mathcal{PT}$-symmetric quantum mechanics" In preparation.

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Bose-Einstein condensates in $\mathcal{PT}$ symmetric double wells

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Abstract

The existence of $\mathcal{PT}$ symmetric wave functions describing Bose-Einstein condensates in realistic one-dimensional and fully three-dimensional double-well setups is investigated theoretically. When particles are removed from one well and coherently injected into the other the external potential is $\mathcal{PT}$ symmetric. We solve the underlying Schrödinger equation which contains a nonlinear term proportional to $|\psi|^2$ - the Gross-Pitaevskii equation - using the time-dependent variational principle (TDVP). We show that the $\mathcal{PT}$ symmetry of the external potential is preserved by both the wave functions and the nonlinear Hamiltonian as long as eigenstates with real eigenvalues are obtained. We find that indeed two branches of real eigenvalues exist up to a critical strength of the nonlinearity, at which the two solutions merge in a branch point. This behaviour is analogous to that found in studies of optical wave guides with loss and gain [1].

A surprising result, however, is that although there also appears a branch of two complex conjugate eigenvalues, which correspond to $\mathcal{PT}$ broken solutions, the latter are not born at the branch point but at a lesser value of the strength of the nonlinearity where they bifurcate from the real eigenvalue branch of the ground state. This implies that there is a range of values of the nonlinearity where two real and two complex eigenvalues coexist. Obviously this is a consequence of the nonlinearity in this $\mathcal{PT}$ symmetric system. It agrees with previous findings in studies of a Bose-Einstein condensate in a $\mathcal{PT}$ symmetric delta-functions double well [2] and of a PT symmetric Bose-Hubbard dimer with loss and gain [3].

The applicability of the TDVP is confirmed by comparing its results with numerically exact solutions in the one-dimensional case. The linear stability analysis and the temporal evolution of condensate wave functions reveal [4] that the $\mathcal{PT}$ symmetric condensates are stable and should be observable in an experiment.

References

[1] S. Klaiman, U. Günther, and N. Moiseyev, Phys. Rev. Lett. 101, 080402 (2008).

[2] H. Cartarius and G. Wunner, preprint arXiv:1203.1885, 2012.

[3] E. M. Graefe, H. J. Korsch, and A. E. Niederle, Phys. Rev. A 82, 013629 (2010).

[4] D. Dast, D. Haag, H. Cartarius, G. Wunner, R. Eichler, and J. Main, Fortschr. Phys., in press, DOI 10.1002/prop.201200080 (2012).

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Quasi-Hermitian theory of quantum catastrophes

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Abstract

The spectra of non-Hermitian operators of observables $F(\lambda), G(\lambda), \ldots$ are assumed all discrete, real and non-degenerate iff $\lambda \in (0, \lambda_{max})$. Inside this interval our observables may be called crypto-Hermitian, i.e., self-adjoint in a "standardized", physical Hilbert space of states $\mathcal{H}^{(S)}(\lambda)$ using an *ad ho*c, $\lambda-$dependent inner product. The name of quantum catastrophe is then assigned to the scenario in which λ leaves the interval $(0, \lambda_{max})$. In the talk we shall address the specific problems encountered during the identification of λ with time. In particular, after a compact summary of the difficulties (related, typically, to the failure of adiabatic approximation) we shall analyze an exactly solvable, cosmology-inspired toy model and demonstrate the compatibility of the standard non-relativistic quantum theory with the various existing classical, non-quantum simulations

1. of the phase transitions resembling the so called Big Bang,
3. of the peculiarities of evolution immediately after the Big-Bang-type quantum catastrophe (illustrative time-dependent metrics will be provided),
4. of certain exceptional, "inflation-like" finite period in the time evolution.

References

[1] M. Znojil, "Time-dependent version of cryptohermitian quantum theory", Phys. Rev. D 78, 085003 (2008).

[2] M. Znojil, "Three-Hilbert-Space Formulation of Quantum Mechanics", SIGMA 5, 001 (2009), arXiv overlay:0901.0700.

[3] M. Znojil, "Quantum catastrophes: a case study", J. Phys. A: Math., Theor., in print, arXiv:1206.6000.

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Posters

Observation of PT phase transition in a simple mechanical system

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Abstract

If a Hamiltonian is PT symmetric, there are two possibilities: Either the eigenvalues are entirely real, in which case the Hamiltonian is said to be in an unbroken-PT-symmetric phase, or else the eigenvalues are partly real and partly complex, in which case the Hamiltonian is said to be in a broken-PT-symmetric phase. As one varies the parameters of the Hamiltonian, one can pass through the phase transition that separates the unbroken and broken phases. This transition has recently been observed in a variety of laboratory experiments. This paper explains the phase transition in a simple and intuitive fashion and then describes an extremely elementary experiment in which the phase transition is easily observed.

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Squeezed coherent states for noncommutative spaces with minimal length uncertainty relations

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Abstract

We provide an explicit construction for Gazeau-Klauder coherent states related to non-Hermitian Hamiltonians with discrete bounded below and nondegenerate eigenspectrum. The underlying spacetime structure is taken to be of a noncommutative type with associated uncertainty relations implying minimal lengths. The uncertainty relations for the constructed states are shown to be saturated in a Hermitian as well as a non-Hermitian setting for a perturbed harmonic oscillator. The computed value of the Mandel parameter dictates that the coherent wavepackets are assembled accord- ing to sub-Poissonian statistics. Fractional revival times, indicating the superposition of classical-like sub-wave packets are clearly identified.

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Feynman-Kleinert method applied to a complex PT-Potential

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Abstract

This method consist to study the simplest form of complex potential that is, $V(x) = (1/4)x^2 + i\lambda x^3$, via a systematic convergent variational perturbation theory for the path integral representation of density matrices

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Pseudo-Hermitian eigenvalue equations in linear-response electronic-structure theory

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Abstract

The electronic excitation energies of molecules or solids are most frequently calculated via the linear response of the electron density to a time-dependent perturbation, as done for example in the random-phase approximation, the Bethe-Salpeter equation or time- dependent density-functional theory [1], and which leads to pseudo-Hermitian eigenvalue equations [2,3,4]. If the response is calculated on a ground state that is properly stable with respect to any perturbation, then one can show that the obtained eigenvalue equation is pseudo-Hermitian with respect to a positive-definite matrix, and the excitation energies are thus real, as they should be. However, in practice, approximations are necessarily made and the approximate ground state is sometimes unstable (e.g., with respect to unphysical spin-symmetry breaking), leading to unexploitable imaginary excitation energies. I will illustrate this on simple examples such as the H2 molecule [5].

References

[1] G. Onida, L. Reining, A. Rubio, Rev. Mod. Phys. **74**, 601 (2002).

[2] A. Mostafazadeh, J. Math. Phys. **43**, 205 (2002).

[3] A. Mostafazadeh, J. Math. Phys. **43**, 2814 (2002).

[4] A. Mostafazadeh, J. Math. Phys. **43**, 3944 (2002).

[5] E. Rebolini, J. Toulouse, A. Savin, submitted.

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