Introduction to *PT* quantum mechanics

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PHHQPXI
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Dirac Hermiticity

$$H = H^{\dagger}$$

($\dagger$ means transpose + complex conjugate)

- guarantees real energy and probability-conserving time evolution
- but … is a **mathematical** axiom and not a **physical** axiom of quantum mechanics

Dirac Hermiticity can be generalized...

The point of this work:

Replace Dirac Hermiticity by *physical* and *weaker* condition of ***PT*** symmetry

P = parity

T = time reversal

Example:

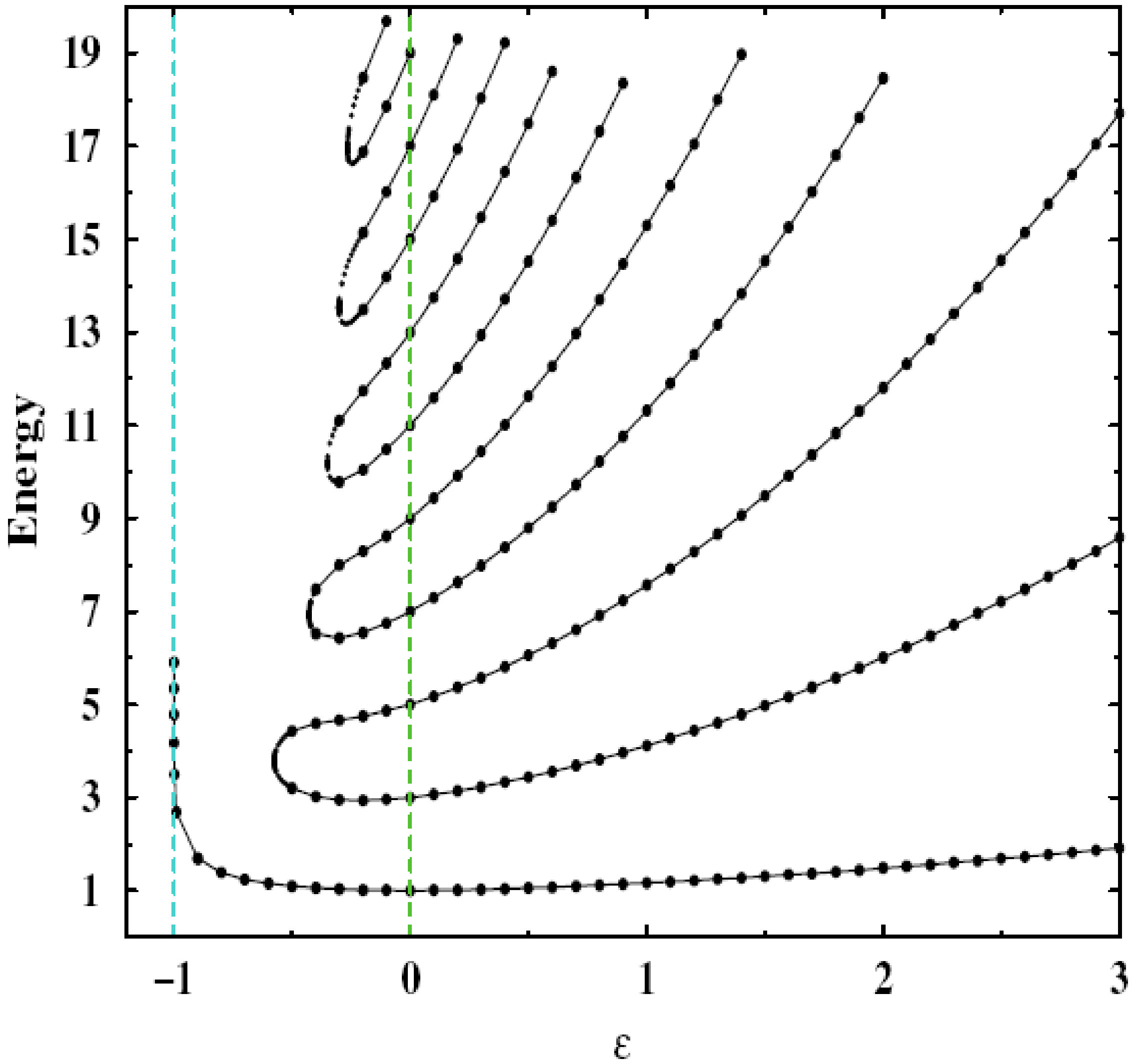
$$\boxed{H = p^2 + ix^3}$$

This Hamiltonian has

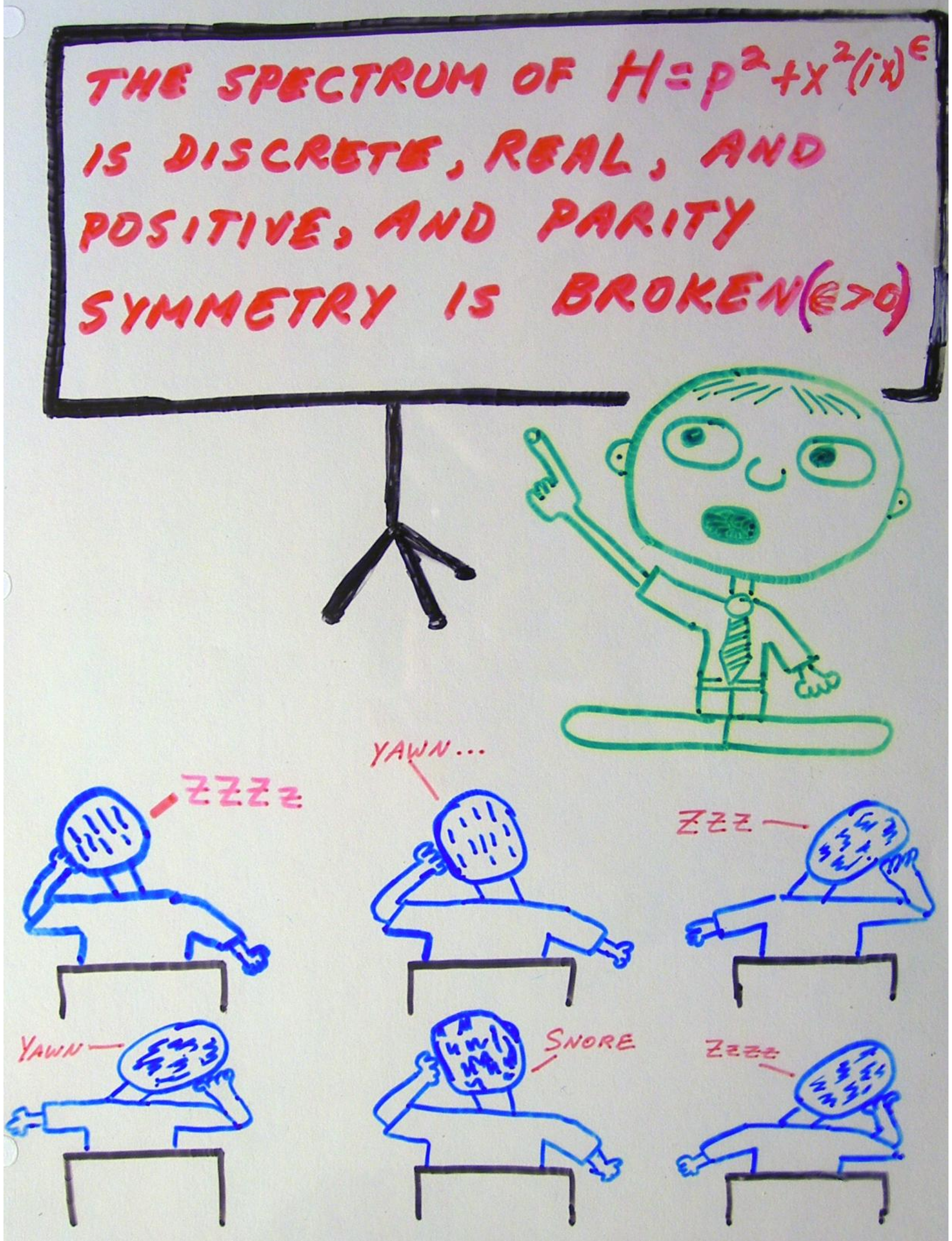
***PT* symmetry!**

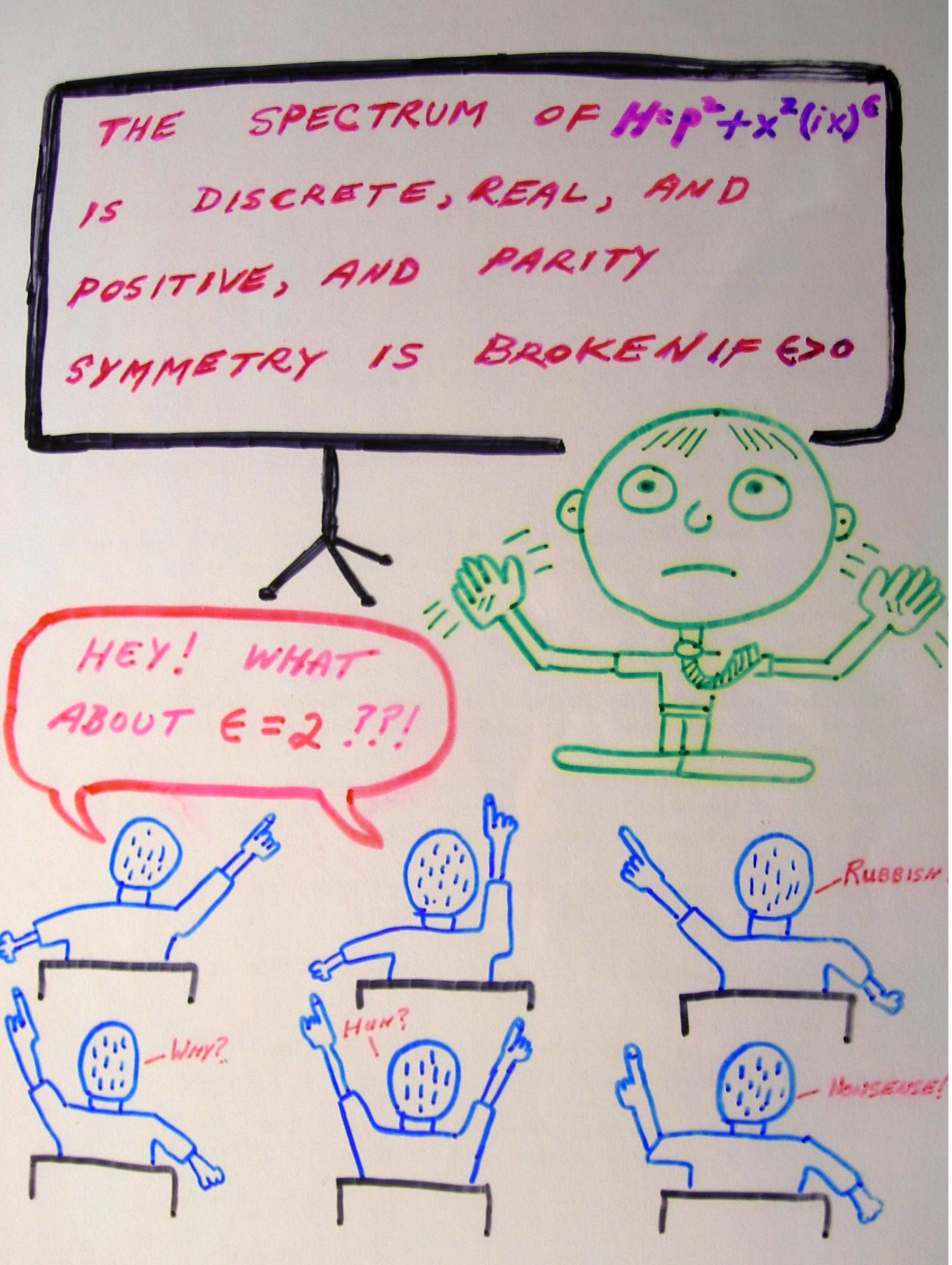
A class of ***PT***-symmetric Hamiltonians:

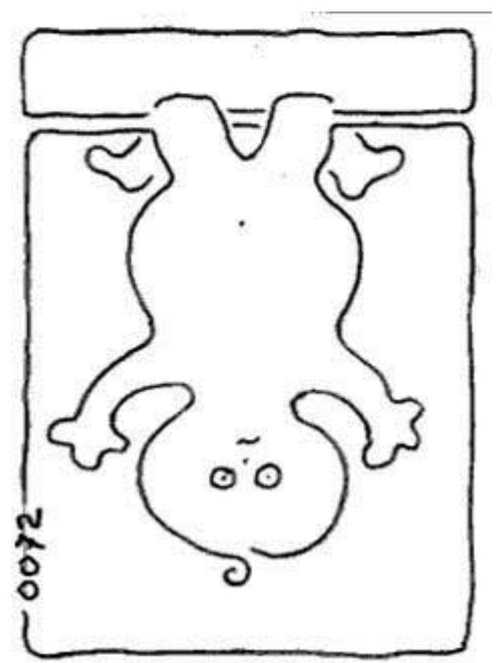
$$\boxed{H = p^2 + x^2(ix)^\varepsilon \quad (\varepsilon \text{ real})}$$



CMB and S. Boettcher
PRL **80**, 5243 (1998)


THE SPECTRUM OF $H=p^2+x^2(ix)^\epsilon$ IS DISCRETE, REAL, AND POSITIVE, AND PARITY SYMMETRY IS BROKEN $(\epsilon>0)$
ZZZZ
YAWN...
ZZZ
YAWN
SNORE
ZZZZ





Upside-down potential with real positive eigenvalues?!

Some of my work

- CMB and S. Boettcher, *Physical Review Letters* **80**, 5243 (1998)
- CMB, D. Brody, H. Jones, *Physical Review Letters* **89**, 270401 (2002)
- CMB, D. Brody, and H. Jones, *Physical Review Letters* **93**, 251601 (2004)
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- CMB, D. Hook, P. Meisinger, Q. Wang, *Physical Review Letters* **104**, 061601 (2010)
- CMB and S. Klevansky, *Physical Review Letters* **105**, 031602 (2010)

PT papers (2008-2010)

- K. Makris, R. El-Ganainy, D. Christodoulides, and Z. Musslimani, *Phyical Review Letters* **100**, 103904 (2008)
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- U. Günther and B. Samsonov, *Physical Review Letters* **101**, 230404 (2008)
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- S. Klaiman, U. Günther, and N. Moiseyev, *Physical Review Letters* **101**, 080402 (2008)
- CMB and P. Mannheim, *Physical Review Letters* **100**, 110402 (2008)

- U. Jentschura, A. Surzhykov, and J. Zinn-Justin, *Physical Review Letters* **102**, 011601 (2009)
- A. Mostafazadeh, *Physical Review Letters* **102**, 220402 (2009)
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- H. Schomerus, *Physical Review Letters* **104**, 233601 (2010)
- S. Longhi, *Physical Review Letters* **105**, 013903 (2010)
- C. West, T. Kottos, T. Prosen, *Physical Review Letters* **104**, 054102 (2010)
- S. Longhi, *Physical Review Letters* **105**, 013903 (2010)
- T. Kottos, *Nature Physics* **6**, 166 (2010)
- C. Ruter, K. Makris, R. El-Ganainy, D. Christodoulides, M. Segev, and D. Kip, *Nature Physics* **6**, 192 (2010)
- CMB, D. Hook, P. Meisinger, Q. Wang, *Physical Review Letters* **104**, 061601 (2010)
- CMB and S. Klevansky, *Physical Review Letters* **105**, 031602 (2010)

PT papers (2011-2012)

- Y. D. Chong, L. Ge, and A. D. Stone, *Physical Review Letters* **106**, 093902 (2011)
- Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, and D. N. Christodoulides, *Physical Review Letters* **106**, 213901 (2011)
- P. D. Mannheim and J. G. O'Brien, *Physical Review Letters* **106**, 121101 (2011)
- L. Feng, M. Ayache, J. Huang, Y. Xu, M. Lu, Y. Chen, Y. Fainman, A. Scherer, *Science* **333**, 729 (2011)

- S. Bittner, B. Dietz, U. Guenther, H. L. Harney, M. Miski-Oglu, A. Richter, F. Schaefer, *Physical Review Letters* **108**, 024101 (2012)
- M. Liertzer, Li Ge, A. Cerjan, A. D. Stone, H. E. Tureci, and S. Rotter, *Physical Review Letters* **108**, 173901 (2012)
- A. Zezyulin and V. V. Konotop, *Physical Review Letters* **108**, 213906 (2012)
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Review articles

- CMB, *Contemporary Physics* **46**, 277 (2005)
- CMB, *Reports on Progress in Physics* **70**, 947 (2007)
- P. Dorey, C. Dunning, and R. Tateo, *Journal of Physics A* **40**, R205 (2007)
- A. Mostafazadeh, *International of Journal of Geometric Methods in Modern Physics* **7**, 1191 (2010)

Developments in *PT* Quantum Mechanics

(Since its ‘official’ beginning in 1998)

- Over fifteen international conferences
- Over 1000 published papers
- Over 122 posts to “***PT* symmeter**” <http://ptsymmetry.net>
  in last 12 months (92 in previous 12 months)
- Lots of experimental results in last two years

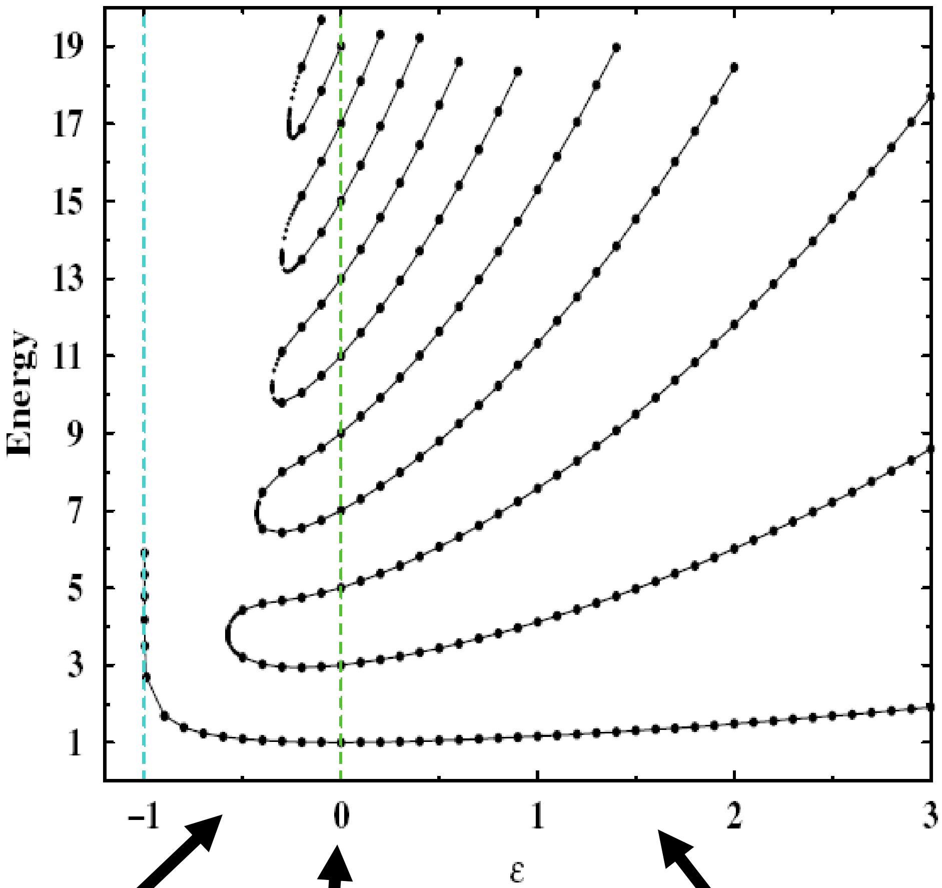
Proving the reality of eigenvalues

Proof is difficult! Uses techniques from conformal field theory and statistical mechanics:

(1) Bethe ansatz
(2) Monodromy group
(3) Baxter T-Q relation
(4) Functional Determinants

[P. Dorey, C. Dunning, and R. Tateo]

$$H = p^2 + x^2(ix)^\varepsilon \quad (\varepsilon \text{ real})$$



Region of ***broken*** ***PT*** symmetry

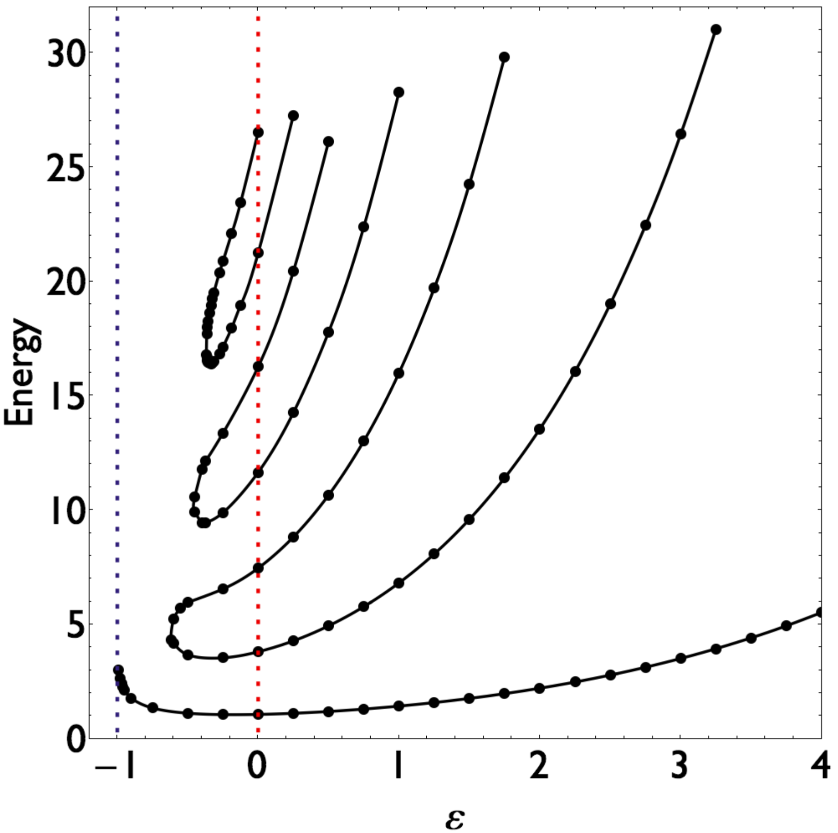
PT **Boundary**

Region of ***unbroken*** ***PT*** symmetry

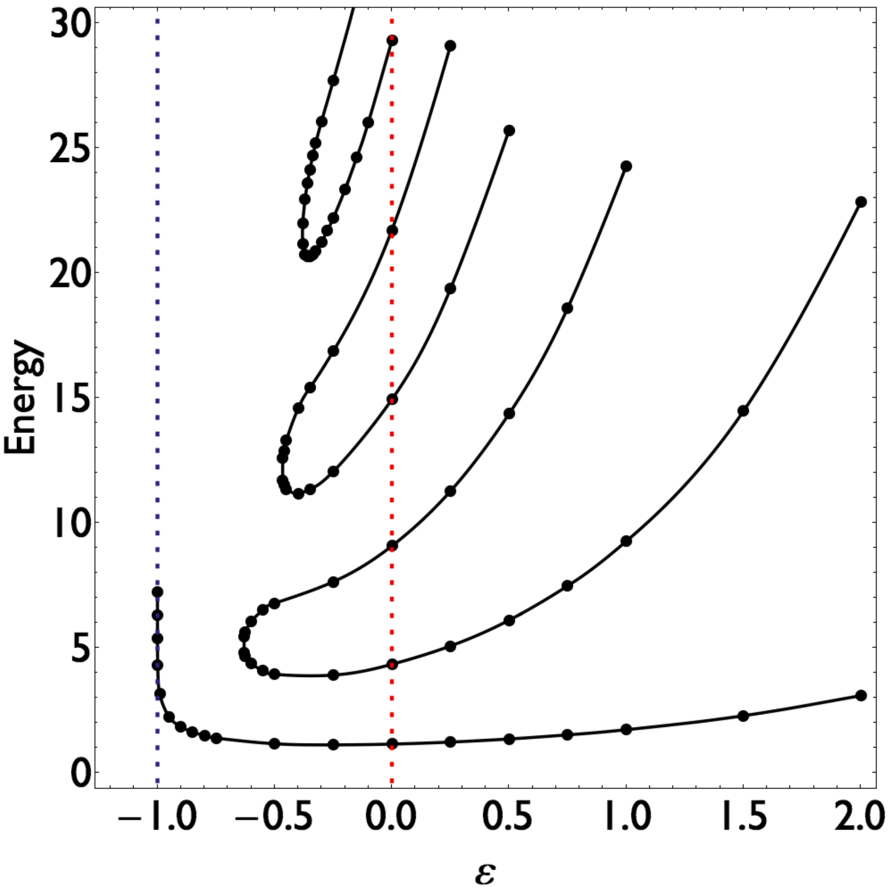
Broken ***P***arro***T*** Unbroken ***P***arro***T***

$$H^{(2n)} = p^{2n} + x^2(ix)^\varepsilon \quad (\varepsilon \text{ real; } n = 1,\, 2,\, 3,\, \ldots)$$

n=2:



n=3:



CMB and D. Hook
Phys. Rev. A **86**, 022113 (2012)

Hermitian Hamiltonians: BORING!

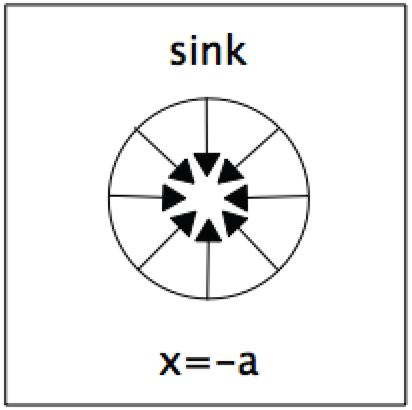
The eigenvalues are always real – nothing interesting happens

PT-symmetric Hamiltonians: ASTONISHING!

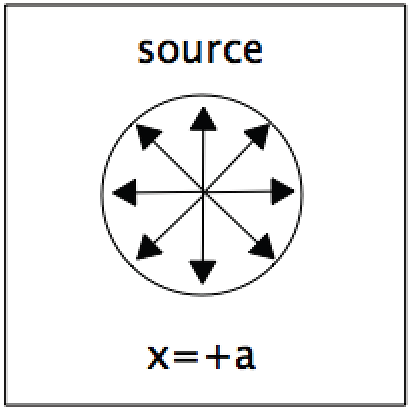
Phase transition between parametric regions of broken and unbroken *PT* symmetry...
Can be observed experimentally!

Intuitive explanation of *PT* phase transition …

Box 1: Loss



Box 2: Gain



$$-i\frac{d}{dt}\phi(t) = H\phi(t)$$

$$H = [E_1] = \left[ae^{i\theta}\right]$$

$$H = [E_2] = \left[ae^{-i\theta}\right]$$

$$\psi(t) = \psi(0)e^{iE_1 t}$$

$$\psi(t) = \psi(0)e^{iE_2 t}$$

Two boxes together as a single system:

$$H = \begin{bmatrix} ae^{i\theta} & 0 \\ 0 & ae^{-i\theta} \end{bmatrix}$$

This Hamiltonian is ***PT*** symmetric,

where ***T*** is complex conjugation and $\mathcal{P} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Couple the boxes together with coupling strength *g*

$$H = \begin{bmatrix} ae^{i\theta} & g \\ g & ae^{-i\theta} \end{bmatrix}$$

Eigenvalues become real if *g* is sufficiently large:

$$g_{\mathrm{crit}}^2 = a^2 \sin^2 \theta$$

Examining CLASSICAL limit of *PT* quantum mechanics provides intuitive explanation of the *PT* transition:

$$H = p^2 + ix^3$$

Source antenna becomes infinitely strong as

$$x \longrightarrow -\infty$$

Sink antenna becomes infinitely strong as

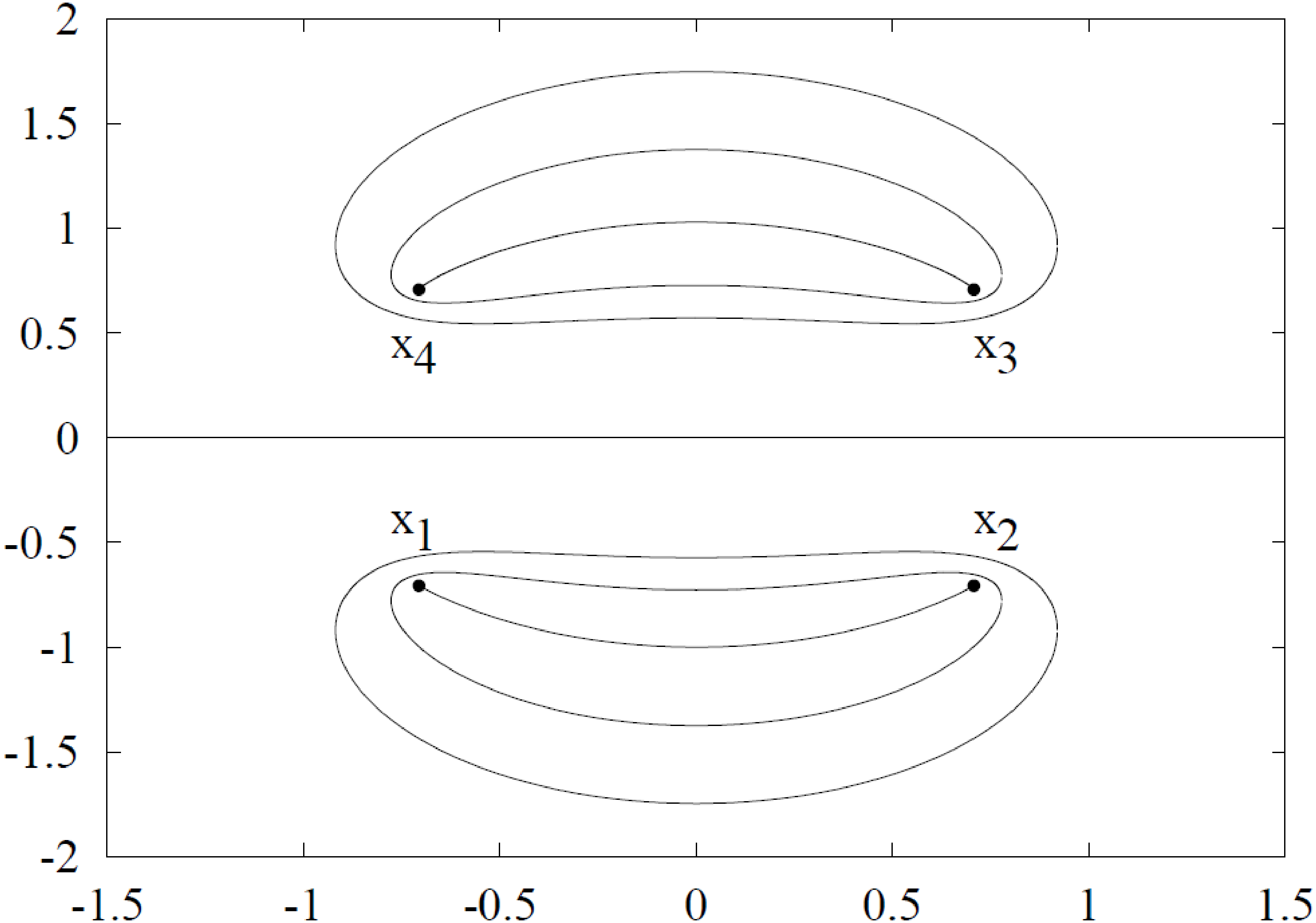
$$x \longrightarrow +\infty$$

Time for classical particle to travel from source to sink:

$$T = \int dt = \int \frac{dx}{p} = \int_{x=-\infty}^{\infty} \frac{dx}{\sqrt{E - ix^3}}$$

$H = p^2 - x^4$

Source and sink localized at + and - infinity



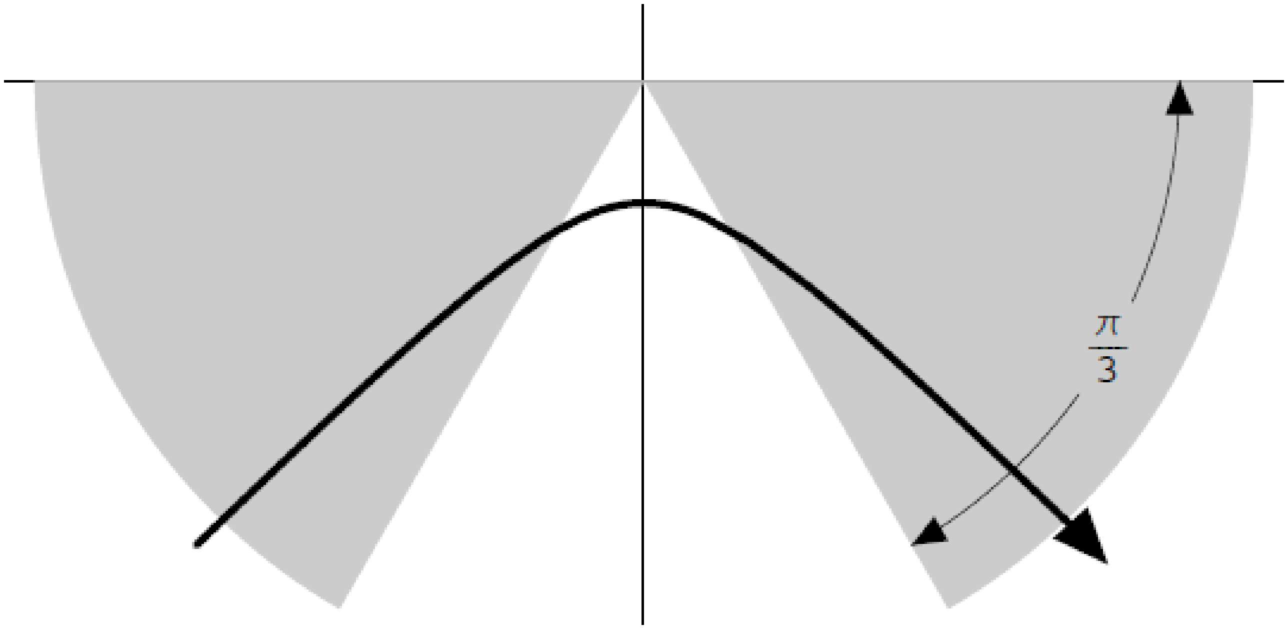
Complex eigenvalue problems and Stokes wedges…

At the quantum level: $H = p^2 - x^4$

Upside down potential

$$H = \frac{1}{2m}p^2 - gx^4$$

$$-\frac{\hbar^2}{2m}\psi''(x) - gx^4\psi(x) = E\psi(x)$$



Step 1: Change path of integration

$$x = -2iL\sqrt{1 + iy/L}$$

fundamental unit of length is $[\hbar^2/(mg)]^{1/6}$

$$L = \lambda\left(\frac{\hbar^2}{mg}\right)^{1/6}$$

λ is an arbitrary positive dimensionless constant

$$-\frac{\hbar^2}{2m}\left(1+\frac{iy}{L}\right)\phi''(y) - \frac{i\hbar^2}{4Lm}\phi'(y) - 16gL^4\left(1+\frac{iy}{L}\right)^2\phi(y) = E\phi(y)$$

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Step 2: Fourier transform

$$\tilde{f}(p) \equiv \int_{-\infty}^{\infty} dy\, e^{-iyp/\hbar} f(y)$$

$$\frac{1}{2m}\left(1-\frac{\hbar}{L}\frac{d}{dp}\right)p^2\tilde{\phi}(p)+\frac{\hbar}{4Lm}p\tilde{\phi}(p)-16gL^4\left(1-\frac{\hbar}{L}\frac{d}{dp}\right)^2\tilde{\phi}(p)=E\tilde{\phi}(p)$$

$$-16gL^2\hbar^2\tilde{\phi}''(p)+\left(-\frac{\hbar p^2}{2mL}+32gL^3\hbar\right)\tilde{\phi}'(p)+\left(\frac{p^2}{2m}-\frac{3p\hbar}{4mL}-16gL^4\right)\tilde{\phi}(p)=E\tilde{\phi}(p)$$

Step 3: Change dependent variable

$$\tilde{\phi}(p) = e^{Q(p)/2}\Phi(p)$$

$$Q(p) = \frac{2L}{\hbar}p - \frac{1}{96gmL^3\hbar}p^3$$

$$-16gL^2\hbar^2\Phi''(p) + \left(-\frac{\hbar p}{4mL} + \frac{p^4}{256gm^2L^4}\right)\Phi(p) = E\Phi(p)$$

Step 4: Rescale *p*

$$p = zL\sqrt{32mg}$$

$$-\frac{\hbar^2}{2m}\Phi''(z) + \left(-\hbar\sqrt{\frac{2g}{m}}\,z + 4gz^4\right)\Phi(z) = E\Phi(z)$$

Result: A pair of exactly isospectral Hamiltonians

$$H = \frac{1}{2m}p^2 - gx^4$$

$$\tilde{H} = \frac{\tilde{p}^2}{2m} - \hbar\sqrt{\frac{2g}{m}}\,z + 4gz^4$$

CMB, D. C. Brody, J.-H. Chen, H. F. Jones , K. A. Milton, and M. C. Ogilvie
Physical Review D **74**, 025016 (2006) [arXiv: hep-th/0605066]

Reflectionless potentials!

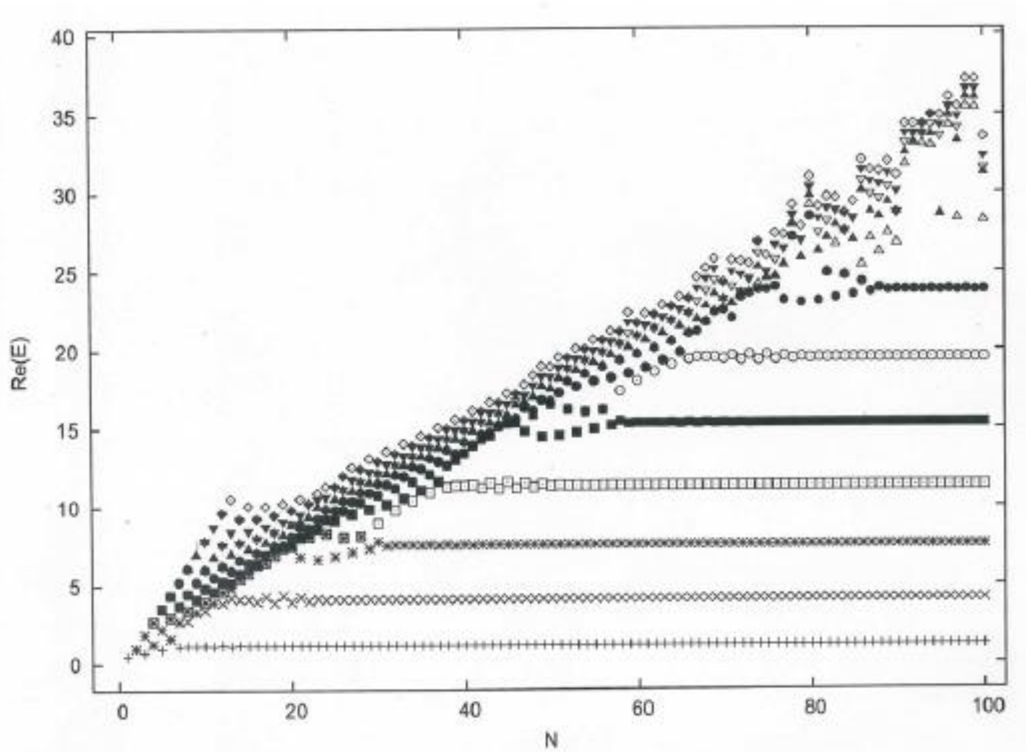
Z. Ahmed, CMB, and M. V. Berry,
J. Phys. A: Math. Gen. **38**, L627 (2005) [arXiv: quant-ph/0508117]

In effect, we are extending conventional classical mechanics and Hermitian quantum mechanics into the complex plane…

Complex plane

How general is the *PT* phase transition?

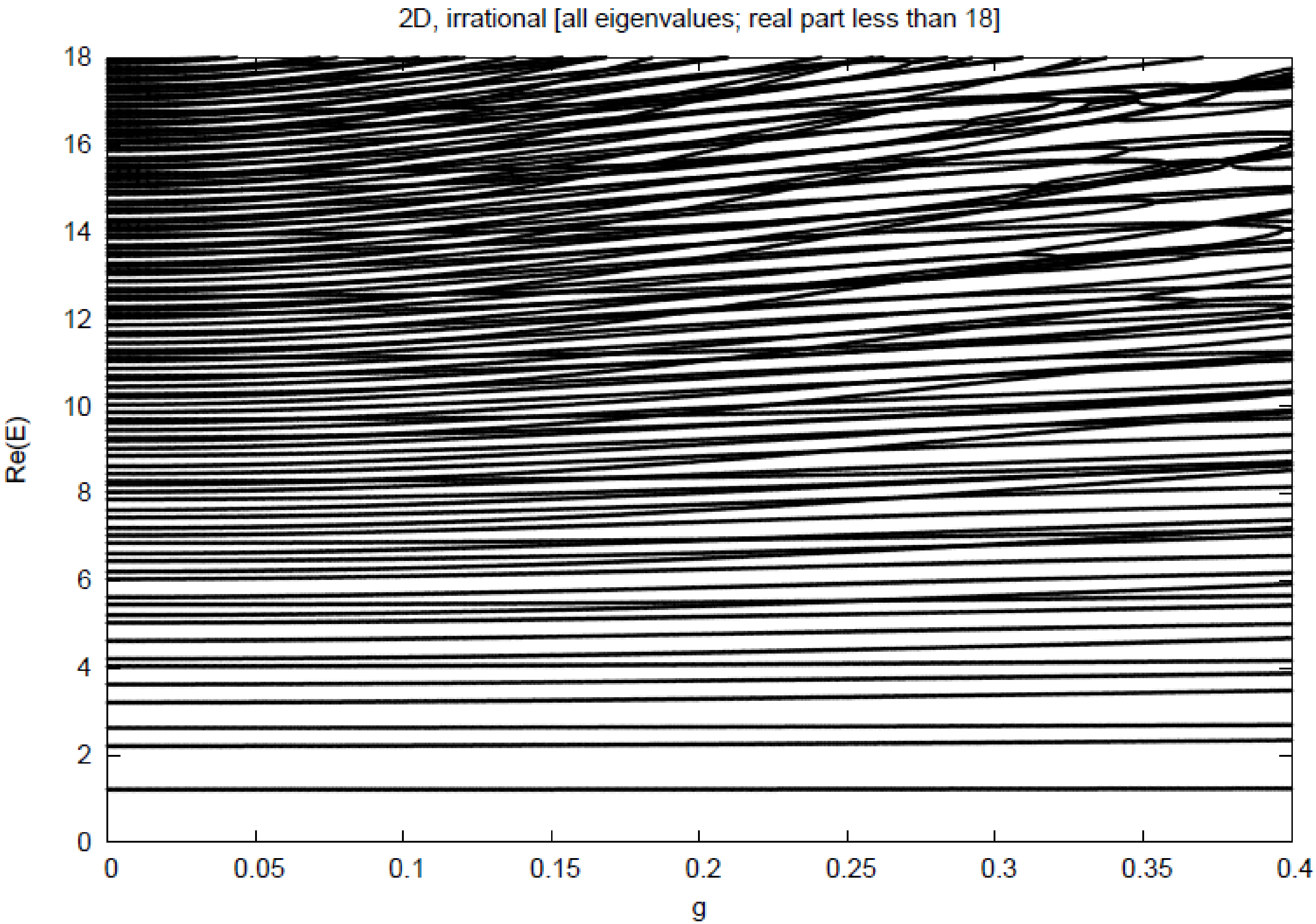
$$H = p^2 + ix^3$$

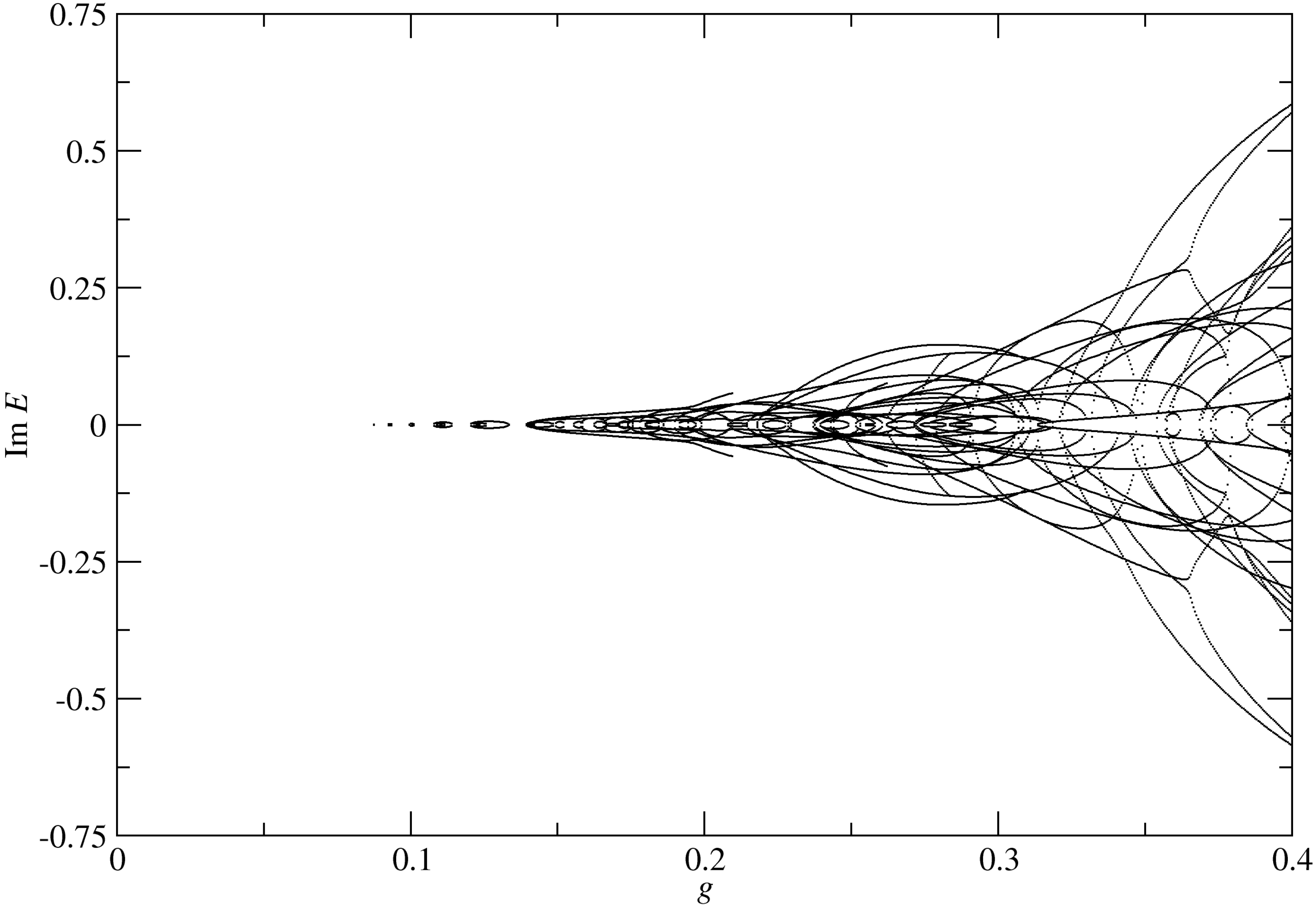


***Implicitly restarted Arnoldi* algorithm**

CMB and D. Weir
[arXiv: quant-ph/1206.5100]

$$H = \frac{1}{2}p^2 + x^2 + \frac{1}{2}q^2 + \frac{1}{2}y^2 + igx^2y$$

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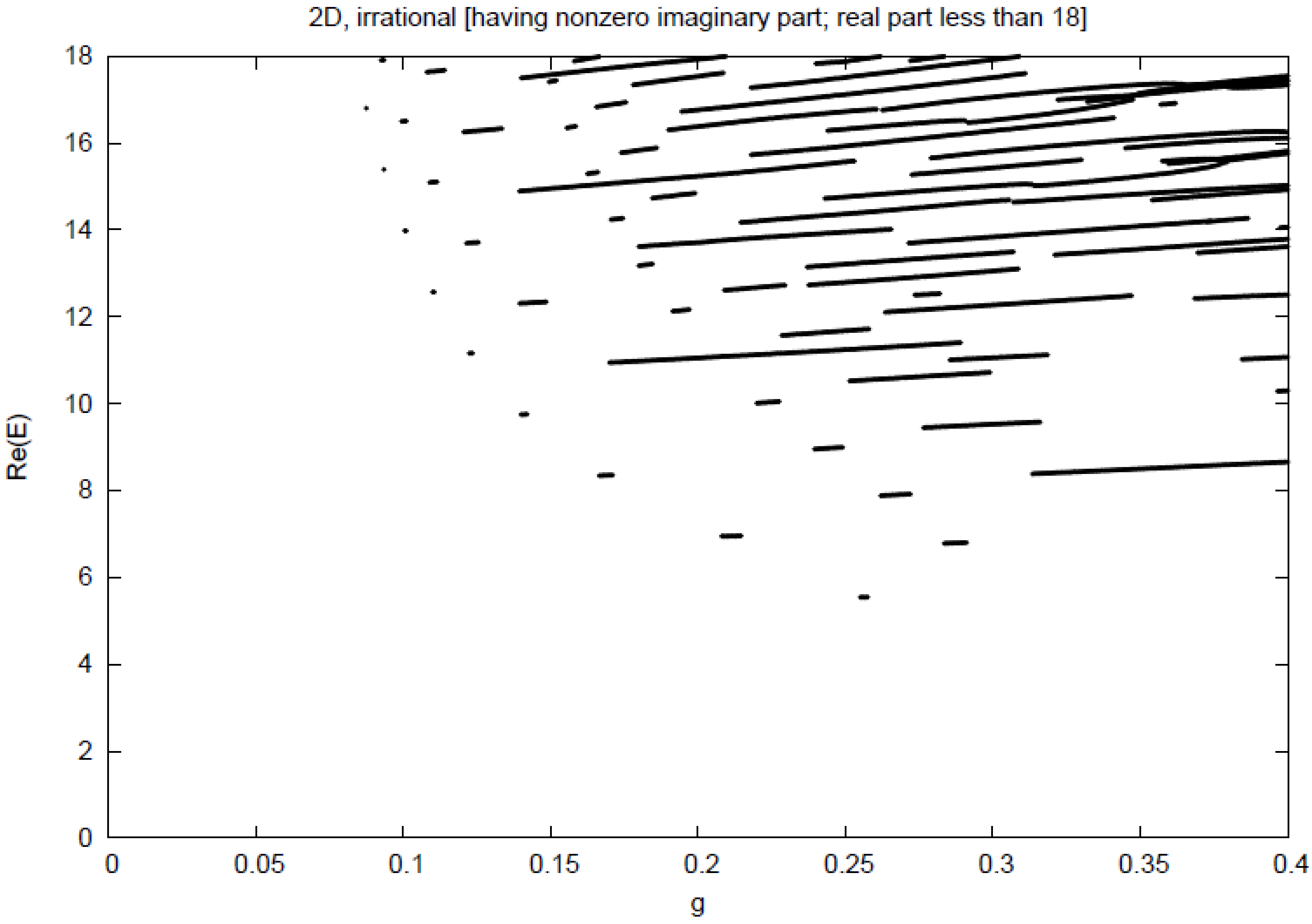


Phase transition at $g = 0.04$

The eigenvalues are real and positive, but is this quantum mechanics?

- **Probabilistic interpretation??**
- **Hilbert space with a positive metric??**
- **Unitarity time evolution??**

The Hamiltonian determines its own adjoint!

$$[\mathcal{C}, \mathcal{PT}] = 0,$$

$$[\mathcal{C}^2 = 1],$$

$$[\mathcal{C}, H] = 0$$

Replace $\dagger$ by $\mathcal{CPT}$

Unitarity

With respect to the ***CPT*** adjoint the theory has UNITARY time evolution.

Norms are strictly positive!
Probability is conserved!

Example: 2 x 2 Non-Hermitian matrix *PT*-symmetric Hamiltonian

$$H = \begin{pmatrix} re^{i\theta} & s \\ s & re^{-i\theta} \end{pmatrix} \qquad (r,\ s,\ \theta \text{ real})$$

$\mathcal{T}$ is complex conjugation and $\mathcal{P} = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$

$$E_{\pm} = r\cos\theta \pm \sqrt{s^2 - r^2\sin^2\theta} \qquad \text{real if } s^2 > r^2\sin^2\theta$$

$$\mathcal{C} = \frac{1}{\cos\alpha}\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$$

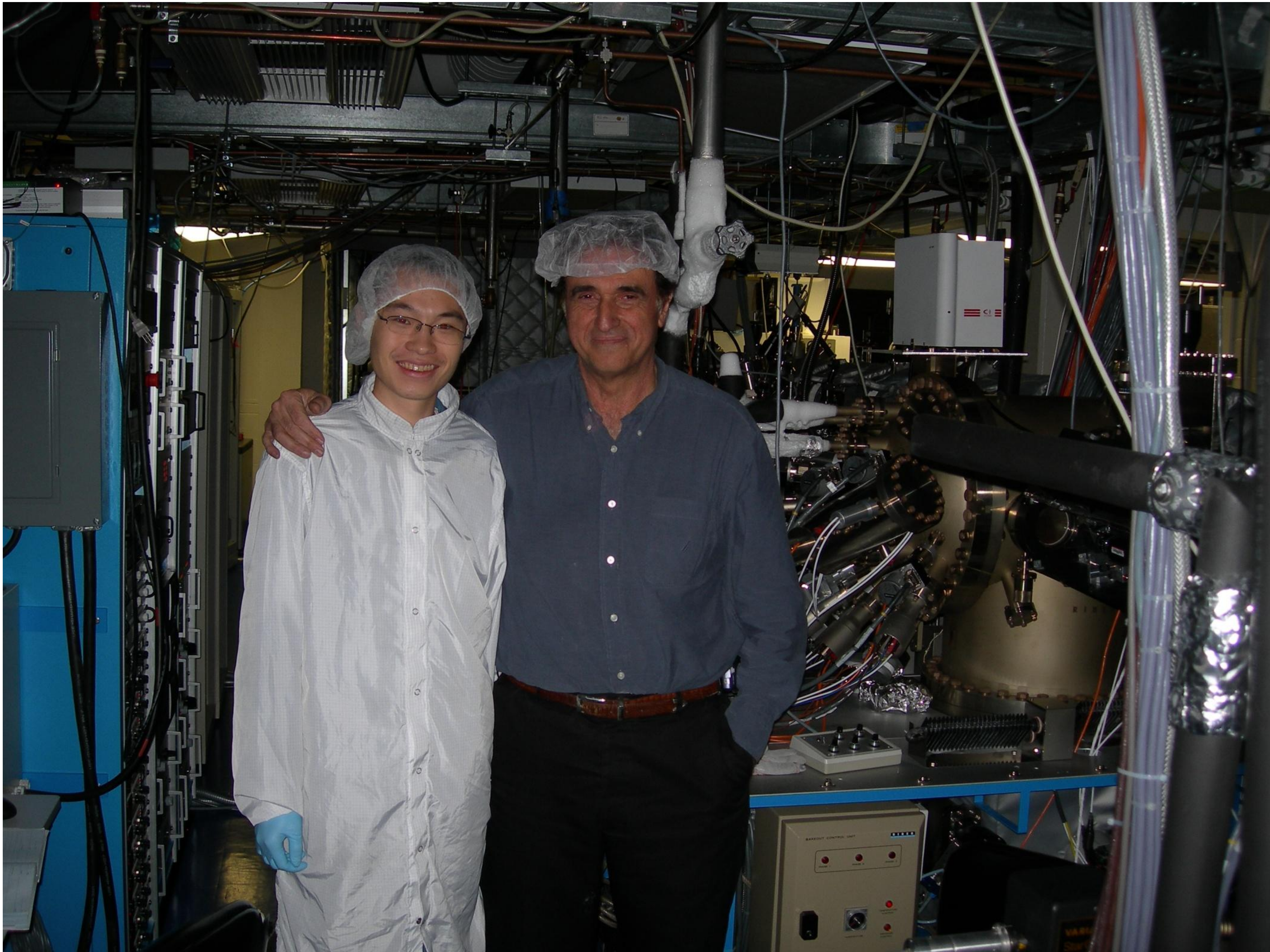
where $\sin\alpha = (r/s)\sin\theta.$

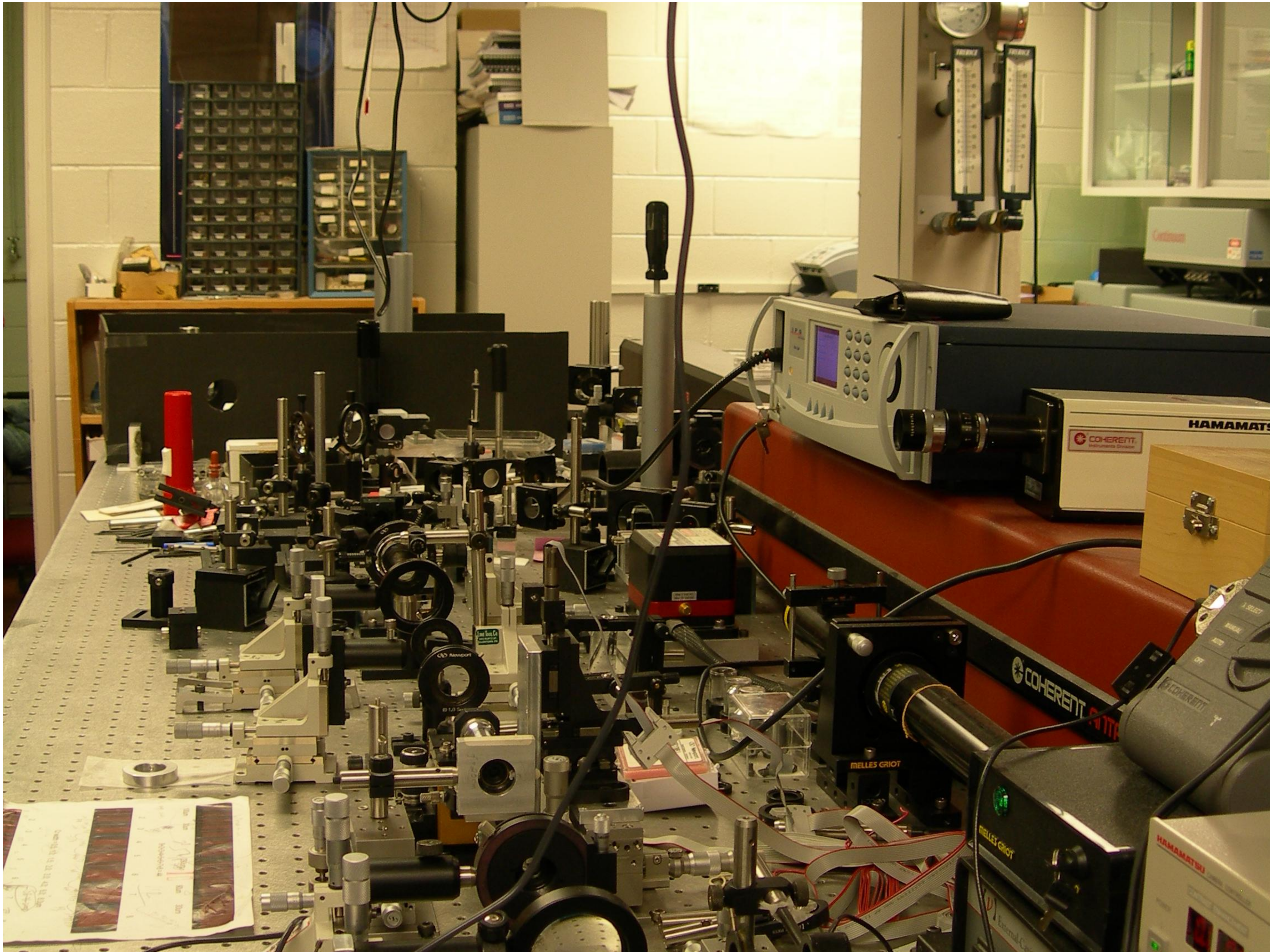
PT–symmetric systems are being observed experimentally!

Laboratory verification using table-top optics experiments!

Observing ***PT*** symmetry using optical wave guides:

- Z. Musslimani, K. Makris, R. El-Ganainy, and D. Christodoulides, *Physical Review Letters* **100**, 030402 (2008)
- K. Makris, R. El-Ganainy, D. Christodoulides, and Z. Musslimani, *Physical Review Letters* **100**, 103904 (2008)
- A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides, *Physical Review Letters* **103**, 093902 (2009)
- C. E. Ruter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, *Nature Physics* **6**, 192 (2010)

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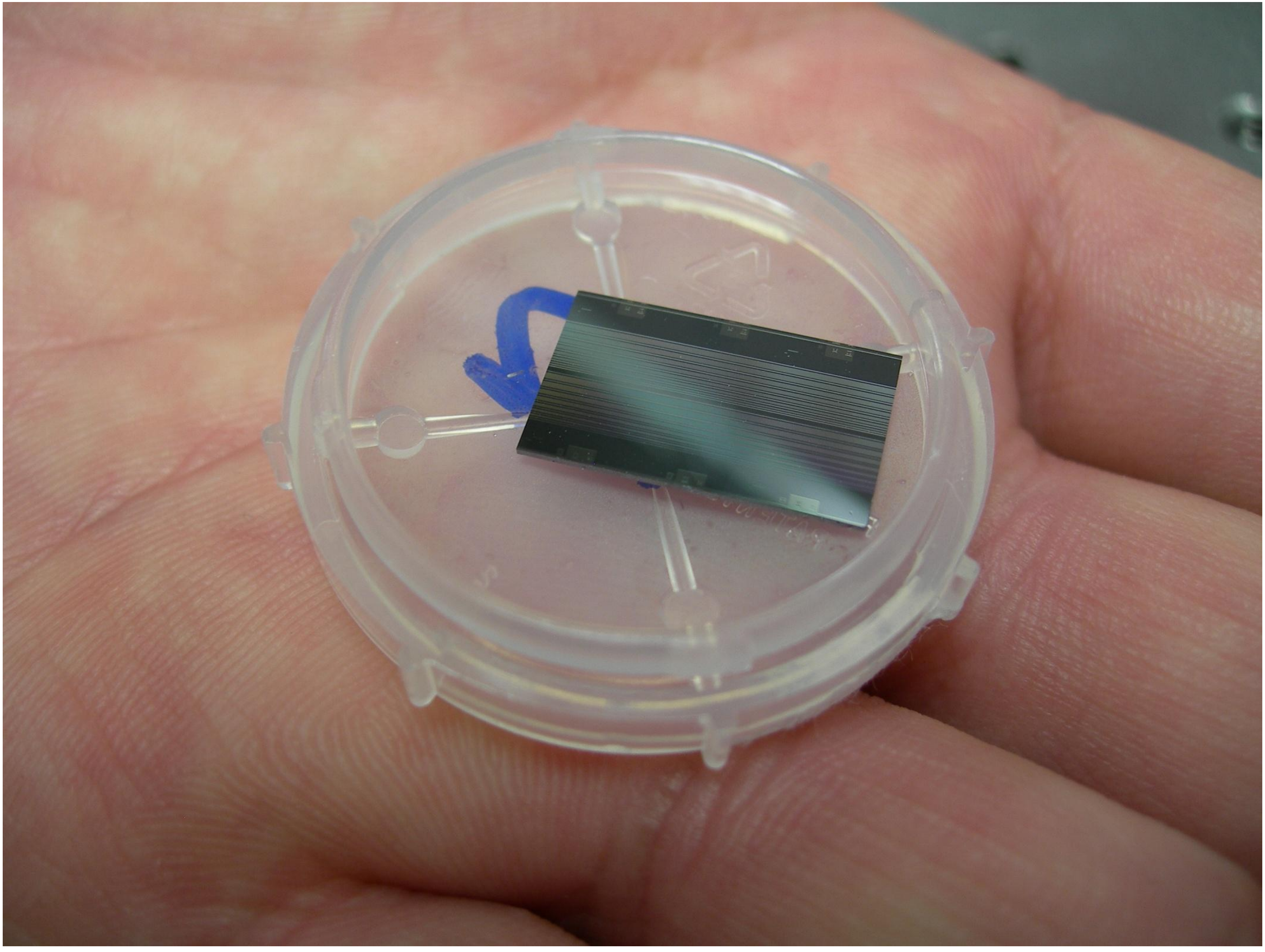
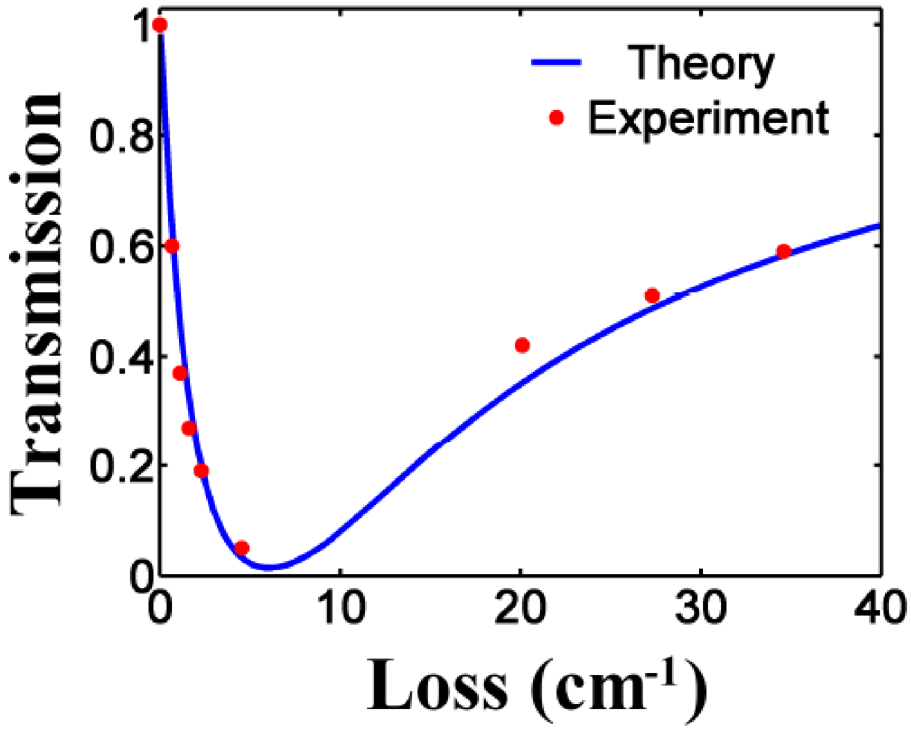


The observed ***PT*** phase transition

Figure 4: Experimental observation of spontaneous passive $\mathcal{PT}$-symmetry breaking. Output transmission of a passive $\mathcal{PT}$ complex system as the loss in the lossy waveguide arm is increased. The transmission attains a minimum at 6 cm^{-1}.



nature physics

LETTERS

PUBLISHED ONLINE: 24 JANUARY 2010 | DOI: 10.1038/NPHYS1515

Observation of parity–time symmetry in optics

Christian E. Rüter[1], Konstantinos G. Makris[2], Ramy El-Ganainy[2], Demetrios N. Christodoulides[2], Mordechai Segev[3] and Detlef Kip[1]*

One of the fundamental axioms of quantum mechanics is associated with the Hermiticity of physical observables[1]. In the case of the Hamiltonian operator, this requirement not only implies real eigenenergies but also guarantees probability conservation. Interestingly, a wide class of non-Hermitian Hamiltonians can still show entirely real spectra. Among these are Hamiltonians respecting parity–time (*PT*) symmetry[2–7]. Even though the Hermiticity of quantum observables was never in doubt, such concepts have motivated discussions on several fronts in physics, including quantum field theories[8], non-Hermitian Anderson models[9] and open quantum systems[10,11], to mention a few. Although the impact of *PT* symmetry in these fields is still debated, it has been recently realized that optics can provide a fertile ground where *PT*-related notions can be implemented and experimentally investigated[12–15]. In this letter we report the first observation of the behaviour of a *PT* optical coupled system that judiciously involves a complex index potential. We observe both spontaneous *PT* symmetry breaking and power oscillations violating left–right symmetry. Our results may pave the way towards a new class of *PT*-synthetic materials with intriguing and unexpected properties that rely on non-reciprocal light propagation and tailored transverse energy flow.

($\varepsilon > \varepsilon_{th}$), the spectrum ceases to be real and starts to involve imaginary eigenvalues. This signifies the onset of a spontaneous *PT* symmetry-breaking, that is, a 'phase transition' from the exact to broken-*PT* phase[7,20].

In optics, several physical processes are known to obey equations that are formally equivalent to that of Schrödinger in quantum mechanics. Spatial diffraction and temporal dispersion are perhaps the most prominent examples. In this work we focus our attention on the spatial domain, for example optical beam propagation in *PT*-symmetric complex potentials. In fact, such *PT* 'optical potentials' can be realized through a judicious inclusion of index guiding and gain/loss regions[7,12–14]. Given that the complex refractive-index distribution $n(x) = n_R(x) + in_I(x)$ plays the role of an optical potential, we can then design a *PT*-symmetric system by satisfying the conditions $n_R(x) = n_R(-x)$ and $n_I(x) = -n_I(-x)$.

In other words, the refractive-index profile must be an even function of position x whereas the gain/loss distribution should be odd. Under these conditions, the electric-field envelope E of the optical beam is governed by the paraxial equation of diffraction[13]:

$$i\frac{\partial E}{\partial z} + \frac{1}{2k}\frac{\partial^2 E}{\partial x^2} + k_0[n_R(x) + in_I(x)]E = 0$$

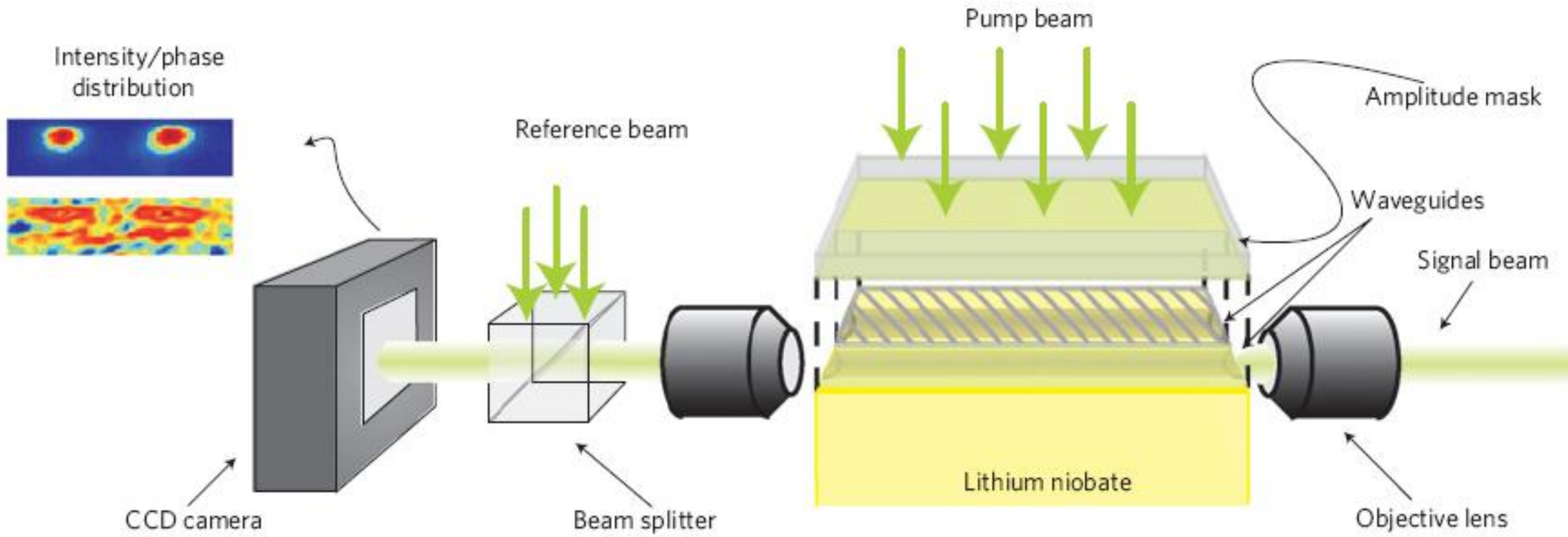


Figure 2 | Experimental set-up. An Ar^+ laser beam (wavelength 514.5 nm) is coupled into the arms of the structure fabricated on a photorefractive $LiNbO_3$ substrate. An amplitude mask blocks the pump beam from entering channel 2, thus enabling two-wave mixing gain only in channel 1. A CCD camera is used to monitor both the intensity and phases at the output.

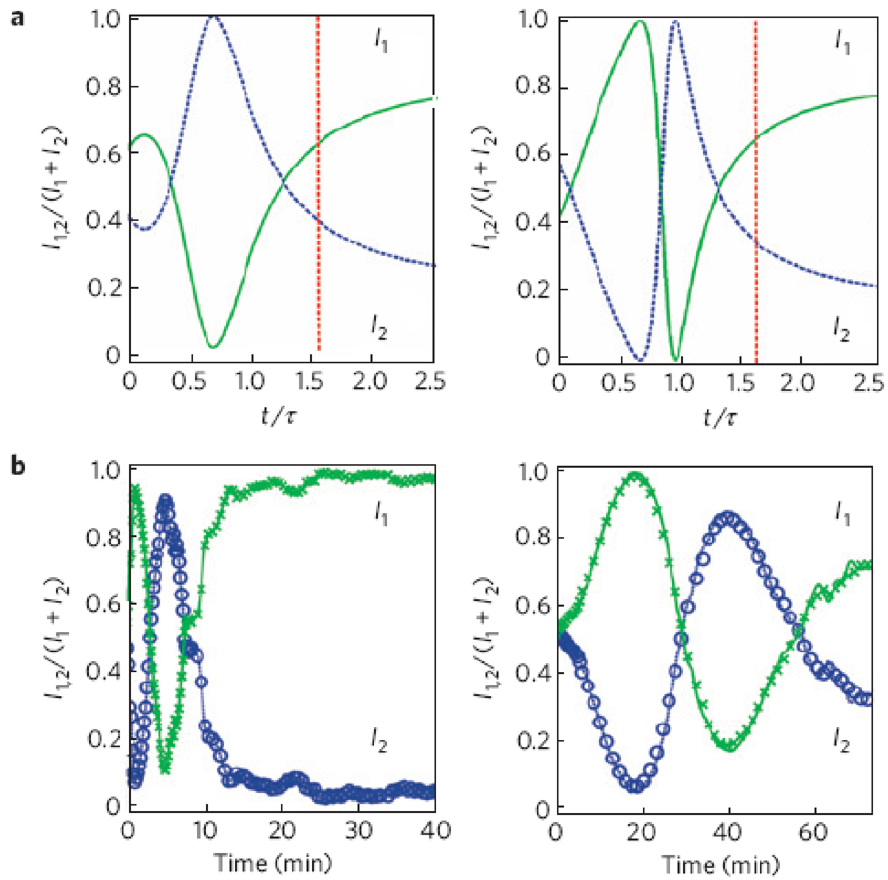


Figure 3 | Computed and experimentally measured response of a *PT*-symmetric coupled system. a, Numerical solution of the coupled equations (1) describing the *PT*-symmetric system. The left (right) panel shows the situation when light is coupled into channel 1 (2). Red dashed lines mark the symmetry-breaking threshold. Above threshold, light is predominantly guided in channel 1 experiencing gain, and the intensity of channels 1 and 2 depends solely on the magnitude of the gain. **b**, Experimentally measured (normalized) intensities at the output facet during the gain build-up as a function of time.

Another experiment...

“Enhanced magnetic resonance signal of spin-polarized Rb atoms near surfaces of coated cells”
K. F. Zhao, M. Schaden, and Z. Wu
Physical Review A **81**, 042903 (2010)

More...

SCIENCE VOL 333 5 AUGUST 2011

Nonreciprocal Light Propagation in a Silicon Photonic Circuit

Liang Feng,[1,2,4]*† Maurice Ayache,[3]* Jingqing Huang,[1,4]* Ye-Long Xu,[2] Ming-Hui Lu,[2] Yan-Feng Chen,[2]† Yeshaiahu Fainman,[3] Axel Scherer[1,4]†

Optical communications and computing require on-chip nonreciprocal light propagation to isolate and stabilize different chip-scale optical components. We have designed and fabricated a metallic-silicon waveguide system in which the optical potential is modulated along the length of the waveguide such that nonreciprocal light propagation is obtained on a silicon photonic chip. Nonreciprocal light transport and one-way photonic mode conversion are demonstrated at the wavelength of 1.55 micrometers in both simulations and experiments. Our system is compatible with conventional complementary metal-oxide-semiconductor processing, providing a way to chip-scale optical isolators for optical communications and computing.

[1]Department of Electrical Engineering, California Institute of Technology, Pasadena, CA 91125, USA. [2]Nanjing National Laboratory of Microstructures, Nanjing University, Nanjing, Jiangsu 210093, China. [3]Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093, USA. [4]Kavli Nanoscience Institute, California Institute of Technology, Pasadena, CA 91125, USA.

Bifurcation Diagram and Pattern Formation of Phase Slip Centers in Superconducting Wires Driven with Electric Currents

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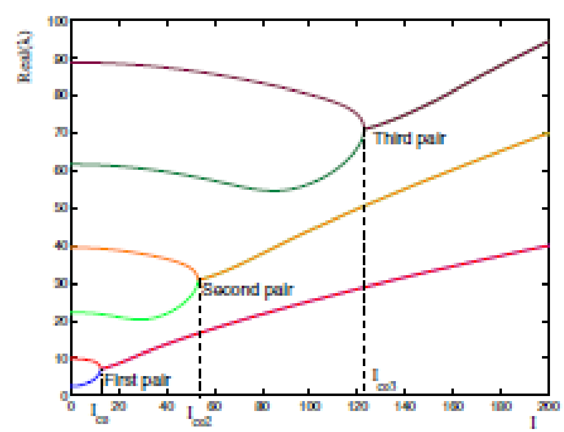
We provide here new insights into the classical problem of a one-dimensional superconducting wire exposed to an applied electric current using the time-dependent Ginzburg-Landau model. The most striking feature of this system is the well-known appearance of oscillatory solutions exhibiting phase slip centers (PSC's) where the order parameter vanishes. Retaining temperature and applied current as parameters, we present a simple yet definitive explanation of the mechanism within this nonlinear model that leads to the PSC phenomenon and we establish where in parameter space these oscillatory solutions can be found. One of the most interesting features of the analysis is the evident collision of real eigenvalues of the associated PT-symmetric linearization, leading as it does to the emergence of complex elements of the spectrum.



$\mathcal{PT}$ Symmetry and Spontaneous Symmetry Breaking in a Microwave Billiard

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We demonstrate the presence of parity-time ($\mathcal{PT}$) symmetry for the non-Hermitian two-state Hamiltonian of a dissipative microwave billiard in the vicinity of an exceptional point (EP). The shape of the billiard depends on two parameters. The Hamiltonian is determined from the measured resonance spectrum on a fine grid in the parameter plane. After applying a purely imaginary diagonal shift to the Hamiltonian, its eigenvalues are either real or complex conjugate on a curve, which passes through the EP. An appropriate basis choice reveals its $\mathcal{PT}$ symmetry. Spontaneous symmetry breaking occurs at the EP.

DOI: 10.1103/PhysRevLett.108.024101 PACS numbers: 05.45.Mt, 02.10.Yn, 11.30.Er

$\mathcal{PT}$-Symmetry Breaking and Laser-Absorber Modes in Optical Scattering Systems

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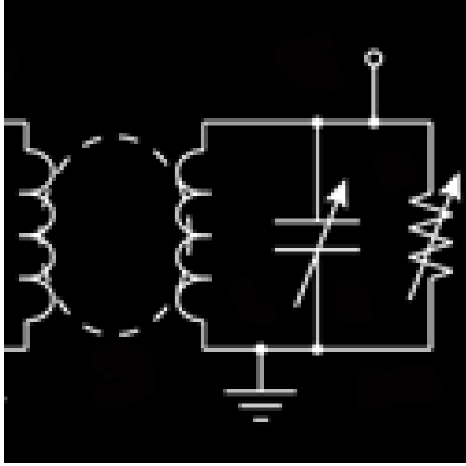
Using a scattering matrix formalism, we derive the general scattering properties of optical structures that are symmetric under a combination of parity and time reversal ($\mathcal{PT}$). We demonstrate the existence of a transition between $\mathcal{PT}$-symmetric scattering eigenstates, which are norm preserving, and symmetry-broken pairs of eigenstates exhibiting net amplification and loss. The system proposed by Longhi [Phys. Rev. A **82**, 031801 (2010).], which can act simultaneously as a laser and coherent perfect absorber, occurs at discrete points in the broken-symmetry phase, when a pole and zero of the S matrix coincide.

DOI: 10.1103/PhysRevLett.106.093902 PACS numbers: 42.25.Bs, 42.25.Hz, 42.55.Ah

APS: Spotlighting exceptional research



J. Schindler *et al.*, Phys. Rev. A (2011)

Experimental study of active *LRC* circuits with *PT* symmetries

Joseph Schindler, Ang Li, Mei C. Zheng, F. M. Ellis, and Tsampikos Kottos

Phys. Rev. A **84**, 040101 (2011)

Published October 13, 2011

Everyone learns in a first course on quantum mechanics that the result of a measurement cannot be a complex number, so the quantum mechanical operator that corresponds to a measurement must be Hermitian. However, certain classes of complex Hamiltonians that are not Hermitian can still have real eigenvalues. The key property of these Hamiltonians is that they are parity-time (*PT*) symmetric, that is, they are invariant under a mirror reflection and complex conjugation (which is equivalent to time reversal).

Hamiltonians that have *PT* symmetry have been used to describe the depinning of vortex flux lines in type-II superconductors and optical effects that involve a complex index of refraction, but there has never been a simple physical system where the effects of *PT* symmetry can be clearly understood and explored. Now, Joseph Schindler and colleagues at Wesleyan University in Connecticut have devised a simple *LRC* electrical circuit that displays directly the effects of *PT* symmetry. The key components are a pair of coupled resonant circuits, one with active gain and the other with an equivalent amount of loss. Schindler *et al.* explore the eigenfrequencies of this system as a function of the “gain/loss” parameter that controls the degree of amplification and attenuation of the system. For a critical value of this parameter, the eigenfrequencies undergo a spontaneous phase transition from real to complex values, while the eigenstates coalesce and acquire a definite chirality (handedness). This simple electronic analog to a quantum Hamiltonian could be a useful reference point for studying more complex applications.

– *Gordon W. F. Drake*

PRL 108, 173901 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Pump-Induced Exceptional Points in Lasers

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We demonstrate that the above-threshold behavior of a laser can be strongly affected by exceptional points which are induced by pumping the laser nonuniformly. At these singularities, the eigenstates of the non-Hermitian operator which describes the lasing modes coalesce. In their vicinity, the laser may turn off even when the overall pump power deposited in the system is increased. Such signatures of a pump-induced exceptional point can be experimentally probed with coupled ridge or microdisk lasers.

Nonlinear Modes in Finite-Dimensional $\mathcal{PT}$-Symmetric Systems

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(Received 7 February 2012; published 24 May 2012)

By rearrangements of waveguide arrays with gain and losses one can simulate transformations among parity-time ($\mathcal{PT}$-) symmetric systems not affecting their pure real linear spectra. Subject to such transformations, however, the nonlinear properties of the systems undergo significant changes. On an example of an array of four waveguides described by the discrete nonlinear Schrödinger equation with dissipation and gain, we show that the equivalence of the underlying linear spectra does not imply similarity of the structure or stability of the nonlinear modes in the arrays. Even the existence of one-parametric families of nonlinear modes is not guaranteed by the $\mathcal{PT}$ symmetry of a newly obtained system. In addition, the stability is not directly related to the $\mathcal{PT}$ symmetry: stable nonlinear modes exist even when the spectrum of the linear array is not purely real. We use a graph representation of $\mathcal{PT}$-symmetric networks allowing for a simple illustration of linearly equivalent networks and indicating their possible experimental design.

ARTICLE

doi:10.1038/nature11298

Parity–time synthetic photonic lattices

Alois Regensburger[1,2], Christoph Bersch[1,2], Mohammad-Ali Miri[3], Georgy Onishchukov[2], Demetrios N. Christodoulides[3] & Ulf Peschel[1]

The development of new artificial structures and materials is today one of the major research challenges in optics. In most studies so far, the design of such structures has been based on the judicious manipulation of their refractive index properties. Recently, the prospect of simultaneously using gain and loss was suggested as a new way of achieving optical behaviour that is at present unattainable with standard arrangements. What facilitated these quests is the recently developed notion of 'parity–time symmetry' in optical systems, which allows a controlled interplay between gain and loss. Here we report the experimental observation of light transport in large-scale temporal lattices that are parity–time symmetric. In addition, we demonstrate that periodic structures respecting this symmetry can act as unidirectional invisible media when operated near their exceptional points. Our experimental results represent a step in the application of concepts from parity–time symmetry to a new generation of multifunctional optical devices and networks.

PT-symmetric system of coupled pendula

$$\begin{aligned} x''(t) + ax'(t) + x(t) + \varepsilon y(t) &= 0 \\ y''(t) - ay'(t) + y(t) + \varepsilon x(t) &= 0 \end{aligned}$$

Best way to have loss and gain:

Set *a*=0

Remove *r* (0 < *r* < 1) of the energy of the *x* pendulum and transfer it to the *y* pendulum.

CMB, B. Berntson, D. Parker, E. Samuel [arXiv: math-ph/1206.4972]

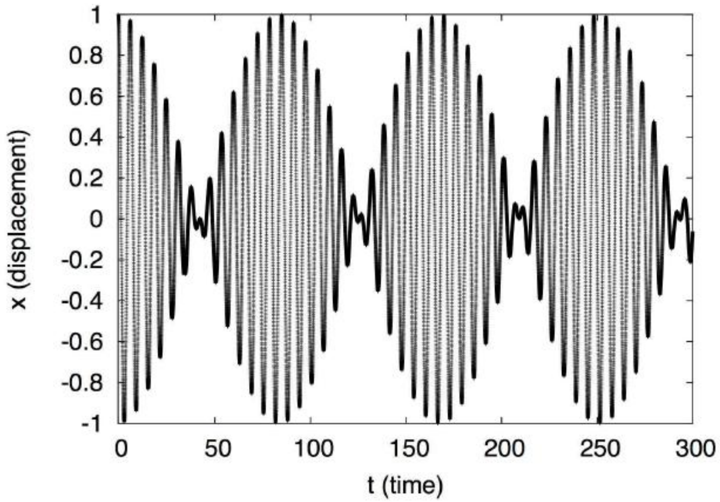
Magnets off

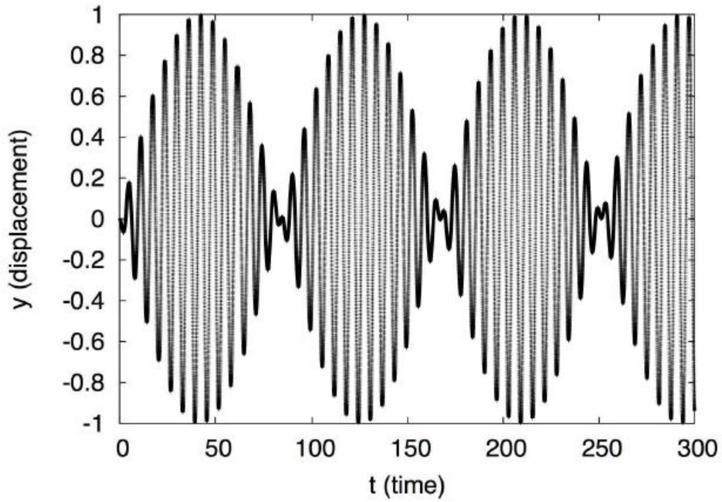
Unbroken *PT*, Rabi oscillations

(pendula in equilibrium)

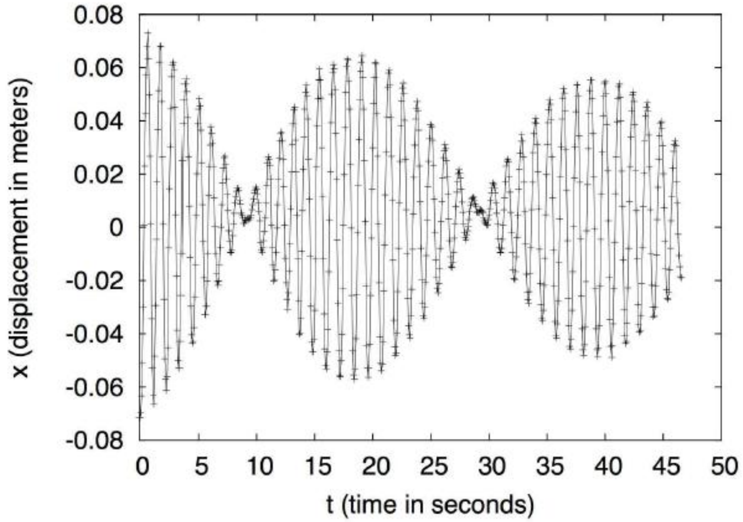
Theory:

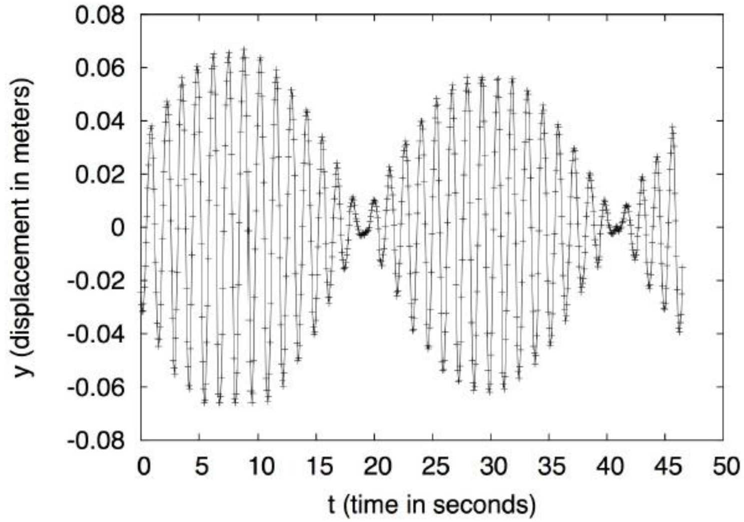
(r=0)





Experiment:



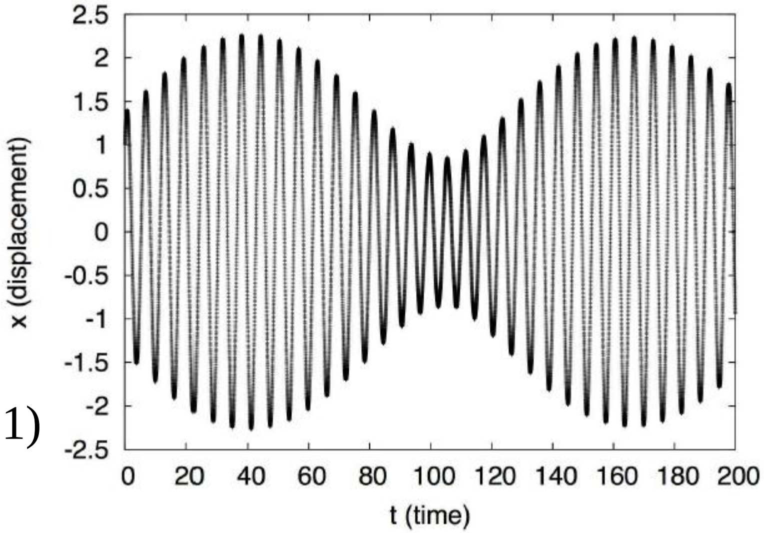


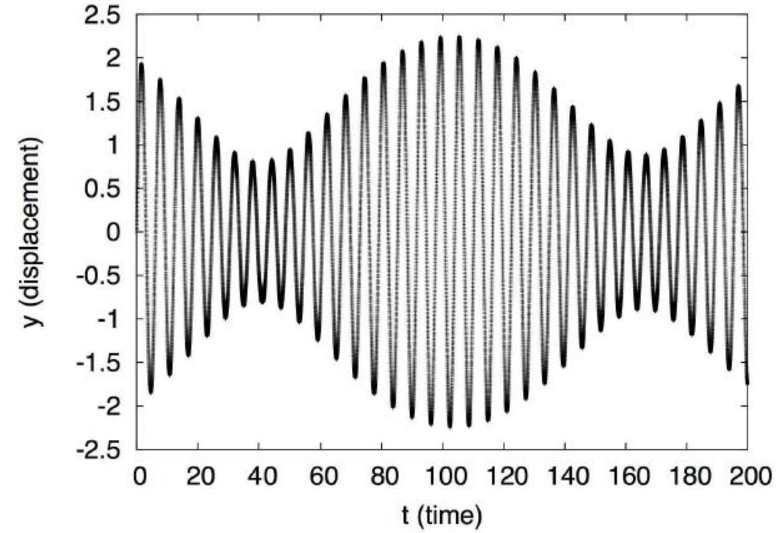
Unbroken *PT* region

Weak magnets, Rabi oscillations (pendula in equilibrium)

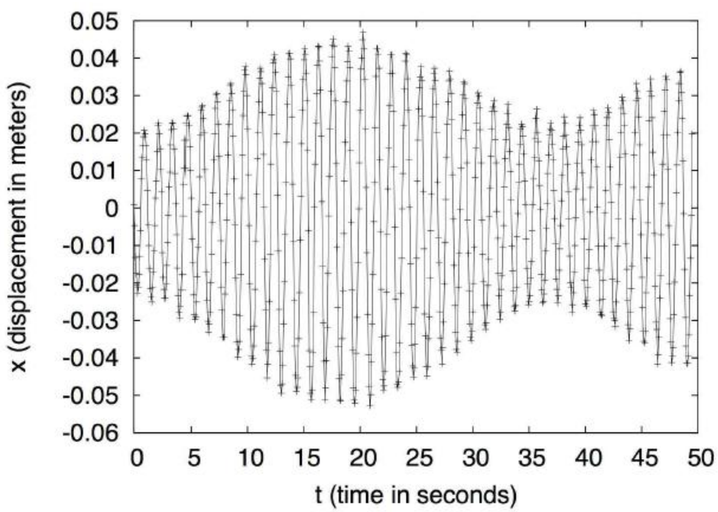
Theory:

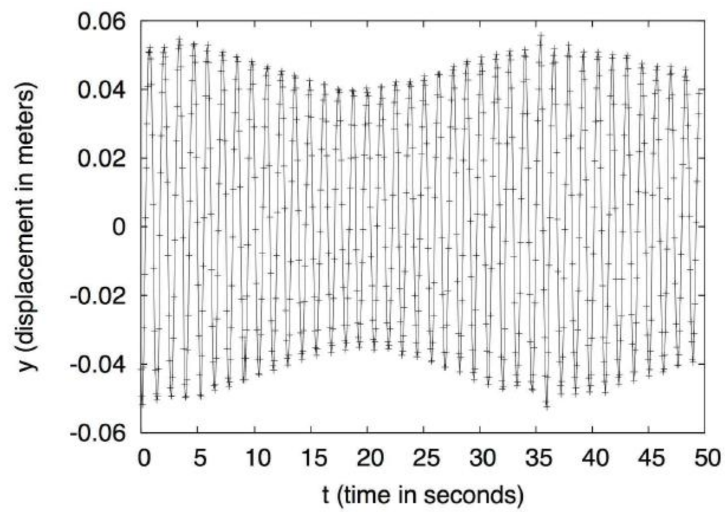
(r=0.01)





Experiment:



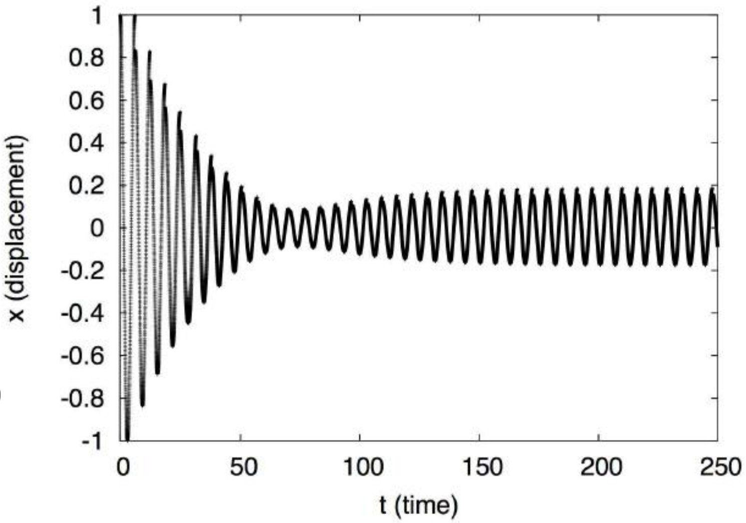


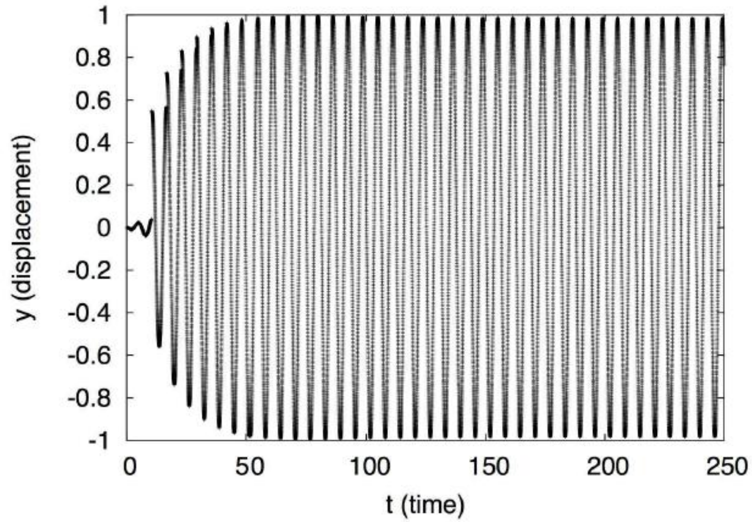
Broken *PT* region

Strong magnets, no Rabi oscillations (pendula out of equilibrium)

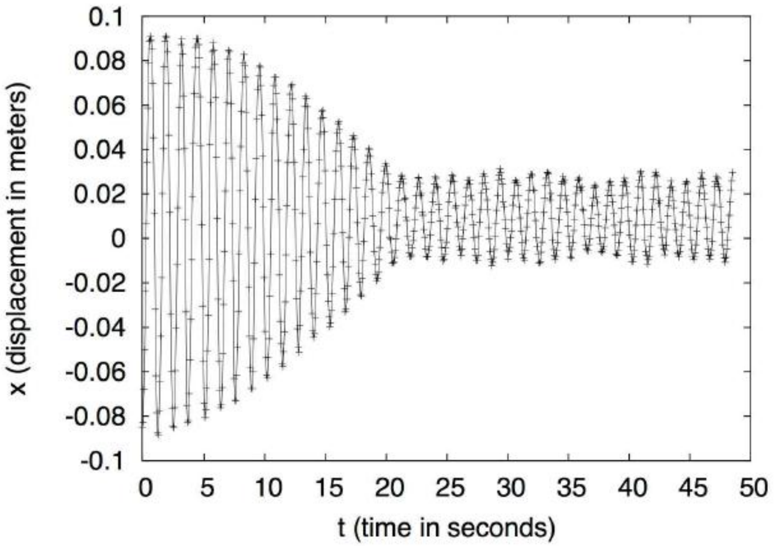
Theory:

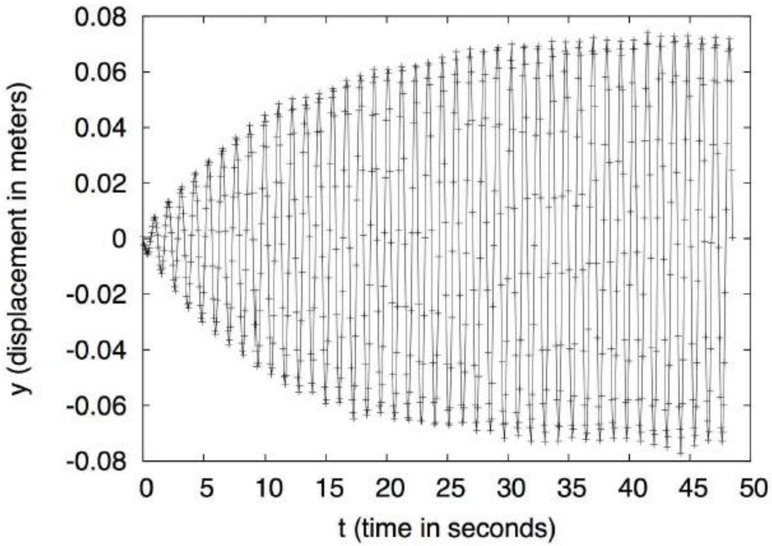
(*r*=0.3)





Experiment:





PT **quantum mechanics is fun! You can re-visit things you already know about ordinary Hermitian quantum mechanics.**

Two examples:

“Ghost Busting: ***PT***-Symmetric Interpretation of the Lee Model”
CMB, S. F. Brandt, J.-H. Chen, and Q. Wang
Phys. Rev. D **71**, 025014 (2005) [arXiv: hep-th/0411064]

“No-ghost Theorem for the Fourth-Order Derivative Pais-Uhlenbeck Oscillator Model”
CMB and P. D. Mannheim
Phys. Rev. Lett. **100**, 110402 (2008) [arXiv: hep-th/0706.0207]

New example:

“Resolution of Ambiguity in the Double-Scaling Limit”
CMB, M. Moshe, and S. Sarkar
[arXiv: hep-th/1206.4943]

Correlated limits

Perturbative solution to a problem:

$$S(\varepsilon,\alpha) \sim \sum_{n=0}^{\infty} a_n(\alpha)\varepsilon^n$$

α tends to a limit as ε approaches 0: $\alpha = \alpha(\varepsilon)$

$$S(\gamma) \text{ is } \textit{entire} \text{ (analytic for all } \gamma)$$

Examples of correlated limits:

(1) Fourier sine series:

$$N \to \infty,\ x \to 0,\ \text{and}\ \gamma \equiv Nx$$

$$\text{Gibbs function}\ G(\gamma) = \mathrm{Si}(2\gamma)$$

(2) Laplace's method for asymptotic expansion of integrals:

$$Z(N) = \int_0^\infty dr\, e^{-NS(r)}$$

Integration by parts:

$$Z(N) \sim e^{-NS(0)} \sum_{k=1}^{\infty} N^{-k} \left[\frac{1}{S'(r)} \frac{d}{dr} \right]^{k-1} \frac{1}{S'(r)} \bigg|_{r=0}$$

Correlated limit: $N \to \infty,\ S'(0) \to 0,\ \text{where } \gamma^2 \equiv N[S'(0)]^2/S''(0)$

$$Z(\gamma) \sim e^{-NS(0)} \exp\left(\gamma^2/4\right) \mathrm{D}_{-1}(\gamma)/\sqrt{NS''(0)}$$

For the special value $\gamma = 0$, $\mathrm{D}_{-1}(0) = \sqrt{\pi/2}$

$$Z(N) \sim e^{-NS(0)} \sqrt{\pi/[2NS''(0)]} \quad (N \to \infty)$$

(3) Transition in a quantum-mechanical wave function between a classically allowed and a classically forbidden region

$$\hbar^2 \phi''(x) = Q(x)\phi(x)$$

$$Q(x) \sim ax \;\; (x \to 0)$$

$$\phi_{\mathrm{WKB}}(x) = \exp\left[\frac{1}{\hbar}\int_0^x ds \sum_{n=0}^{\infty} \hbar^n S_n(s)\right] \quad (\hbar \to 0)$$

$$\hbar \to 0,\, x \to 0$$

$$\gamma = a^{1/2}x^{3/2}/\hbar \;\; \text{held fixed}$$

$$\phi(\gamma) = C\mathrm{Ai}(\gamma)$$

Uncorrelated large-N series for quantum field theory in zero dimensions: The partition function

$$Z = \int d^{N+1}x \exp\left[-\frac{1}{2}\sum_{n=1}^{N+1} x_n^2 - \frac{\lambda}{4}\left(\sum_{n=1}^{N+1} x_n^2\right)^2\right]$$

O$(N+1)$ symmetry

$$\sum_{k=0}^{\infty} a_k N^{-k}$$

Correlated limit (double-scaling limit):

$$N \rightarrow \infty \text{ and } g \rightarrow g_{\mathrm{crit}} = -1/4$$

$$\text{with } \gamma \equiv NG^3/2$$

(Two quadratic saddle points fuse into a cubic saddle point)

Result:

$$Z \sim \mathcal{A}_{N+1} e^{NL(\sqrt{2})} 2^{-1/6} \pi N^{-1/3} \mathrm{Bi}(\gamma^{2/3}) e^{-2\gamma/3}$$

This is *invalid* because $g < 0$

PT-symmetric reformulation:

$$L = \frac{1}{2}\sum_{j=1}^{N+1} x_j^2 + \frac{\lambda i^{\varepsilon}}{2+\varepsilon}\left(\sum_{j=1}^{N+1} x_j^2\right)^{1+\varepsilon/2}$$

Only works when the dimension ***N***+1 is odd!

"The shortest path between two truths in the real domain passes through the complex domain."

-- Jacques Hadamard

The Mathematical Intelligencer **13** (1991)

Possible fundamental applications:

1. ***PT*** Higgs model: $-g\phi^4$ theory is asymptotically free, stable, conformally invariant, and has $\langle\phi\rangle \neq 0$

2. ***PT*** QED $eA_\mu J^\mu$ like a theory of magnetic charge, asymptotically free, opposite Coulomb force

3. ***PT*** gravity $G\phi_{\mu\nu}T^{\mu\nu}$ has a repulsive force

THE END!