NON-UNITARY EVOLUTION AND NON HERMITIAN HAMILTONIANS IN DECOHERENCE OF CLOSED SYSTEMS

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ORGANIZATION OF THE TALK

- ➢ The sense in which we understand the decoherence.
- ➢ Brief explanation of decoherence as proposed by Zurek.
- ➢ The conceptual problems in decoherence theory.
- ➢ The introduction of a particular non-Hermitian effective Hamiltonian to solve one of this problems.
- ➢ Definition of decoherence in closed systems.

MEAN VALUE

A classical set of events is associated with probabilities.

$$\Omega = \{1,2,3,4,5,6\}$$

$$P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = 1/6$$

We can compute the mean value.

$$\langle \Omega \rangle = 1\frac{1}{6} + 2\frac{1}{6} + 3\frac{1}{6} + 4\frac{1}{6} + 5\frac{1}{6} + 6\frac{1}{6}$$

In general:

$$\Omega = \{\omega_1, \omega_2, \omega_3, \ldots\}$$

$$\langle \Omega \rangle = \sum_i \omega_i P_i = \omega_1 P_1 + \omega_2 P_2 + \omega_3 P_3 + \ldots$$

QUANTUM MEAN VALUE

The quantum mean value of the operator *O* is computed as:

$$\left\langle \hat{O} \right\rangle_{\rho} = Tr\left(\hat{O}\hat{\rho}\right) = \sum_{i} o_{ij}\rho_{ji}$$

That is:

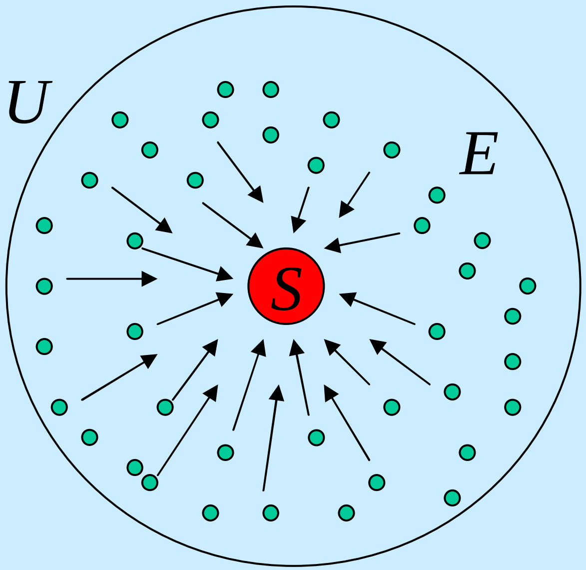
$$\left\langle \hat{O} \right\rangle_{\hat{\rho}} = \sum_{i} o_{ii}\rho_{ii} + \sum_{i \neq j} o_{ij}\rho_{ji}$$

- Superposition
- Interference

It is not possible to interpret the state as a statistical ensemble.

DECOHERENCE PROGRAM

Approach called environment-induced decoherence (EID).



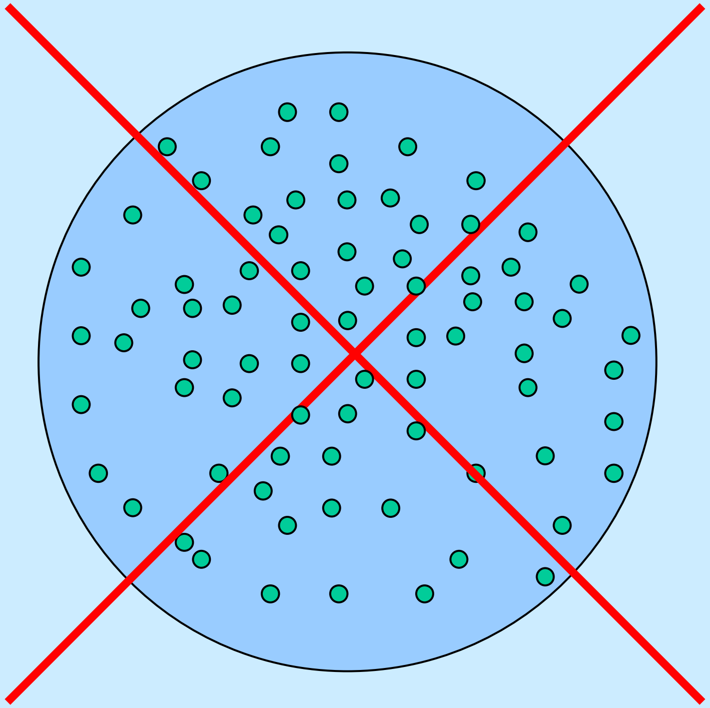
As usual we will consider a closed system U and we will define two subsystems: S, the "proper or open system", and E, the environment. The open system S is considered in interaction with the environment E, and we study the reduced state.

$$\frac{d}{dt}\rho_S(t) = F(\rho_S(t))$$

$$\lim_{t\to\infty}\rho_S(t) = \rho_{S*} \textit{ diagonal} \longleftarrow \text{decoherence}$$

PROBLEMS OF EID

1. It can not be applied to closed systems (NO ENVIRONMENT)



In particular, it can not be applied to the universe as a whole.

DECOHERENCE

In classical systems we have to eliminate the cross terms.

$$\langle O \rangle_\rho = \sum_i o_{ii}\rho_{ii} + \sum_{i\neq j} o_{ij}\rho_{ji}$$

DECOHERENCE

$$\langle O \rangle_\rho = \sum_i o_{ii}\rho_{ii}$$

INTERPRETATION

$$\langle O \rangle_\rho = \sum_i o_i P_i$$

MEAN VALUES APPROACH

States: when the state is diagonal, the interference terms disappear from the mean values of **all** the observables.

decoherence

$\hat{\rho}$ *diagonal* $\longrightarrow$ $\forall \hat{O}, \left\langle \hat{O} \right\rangle_{\hat{\rho}} = \sum_i o_{ii} \rho_{ii}$

Mean Values: If the interference terms disappear from the mean values of **all** the observables, then ρ is diagonal.

decoherence

$\forall \hat{O}, \left\langle \hat{O} \right\rangle_{\hat{\rho}} = \sum_i o_{ii} \rho_{ii}$ $\longrightarrow$ $\hat{\rho}$ *diagonal*

MEAN VALUES APPROACH

If our attention is restricted only to **some** observables instead of all of them, there is no need for the state to be diagonal

no − decoherence

$$\hat{\rho}\ \textit{no}-\textit{diagonal} \longrightarrow \left\langle \hat{O}_R \right\rangle_{\hat{\rho}} = \sum_i o_{ii}\,\rho_{ii}$$

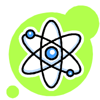
decoherence

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}} = \sum_i o_{ii}\,\rho_{ii} \nrightarrow \cancel{\hat{\rho}\ \textit{diagonal}}$$

A difference appears between these two perspectives: the approach which emphasizes the states is more restrictive.

IRREVERSIBILITY AND DECOHERENCE

A physical system has an associated state operator ρ(t).

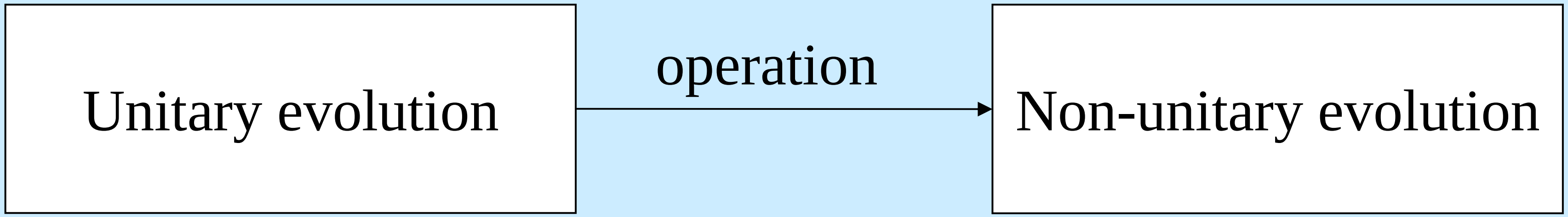
 $\longrightarrow$ $\hat{\rho}_0$

$$\hat{\rho}(t) = e^{-i\frac{\hat{H}}{\hbar}t}\,\hat{\rho}_0\, e^{i\frac{\hat{H}}{\hbar}t}$$

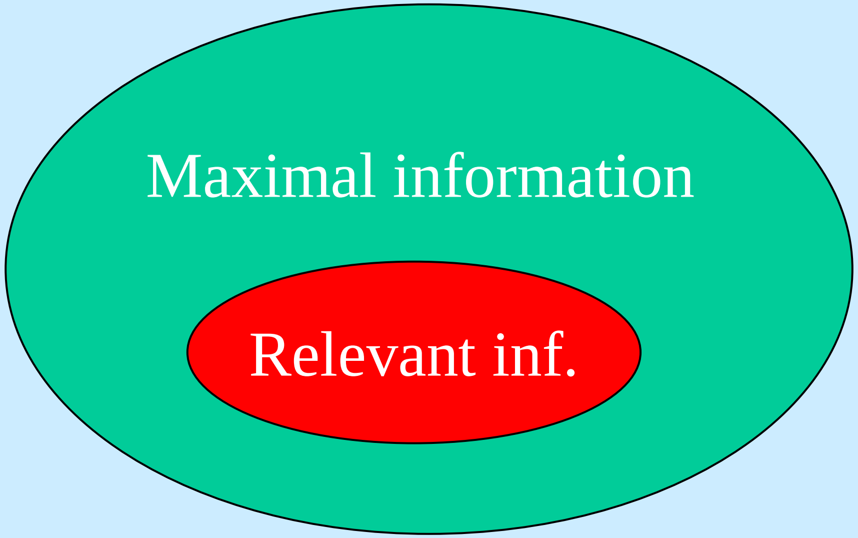
|The state evolves in a unitary way and this prevents it to reach the equilibrium.

This means that the use of some kind of non-unitary evolution is needed to explain the arrival to equilibrium.

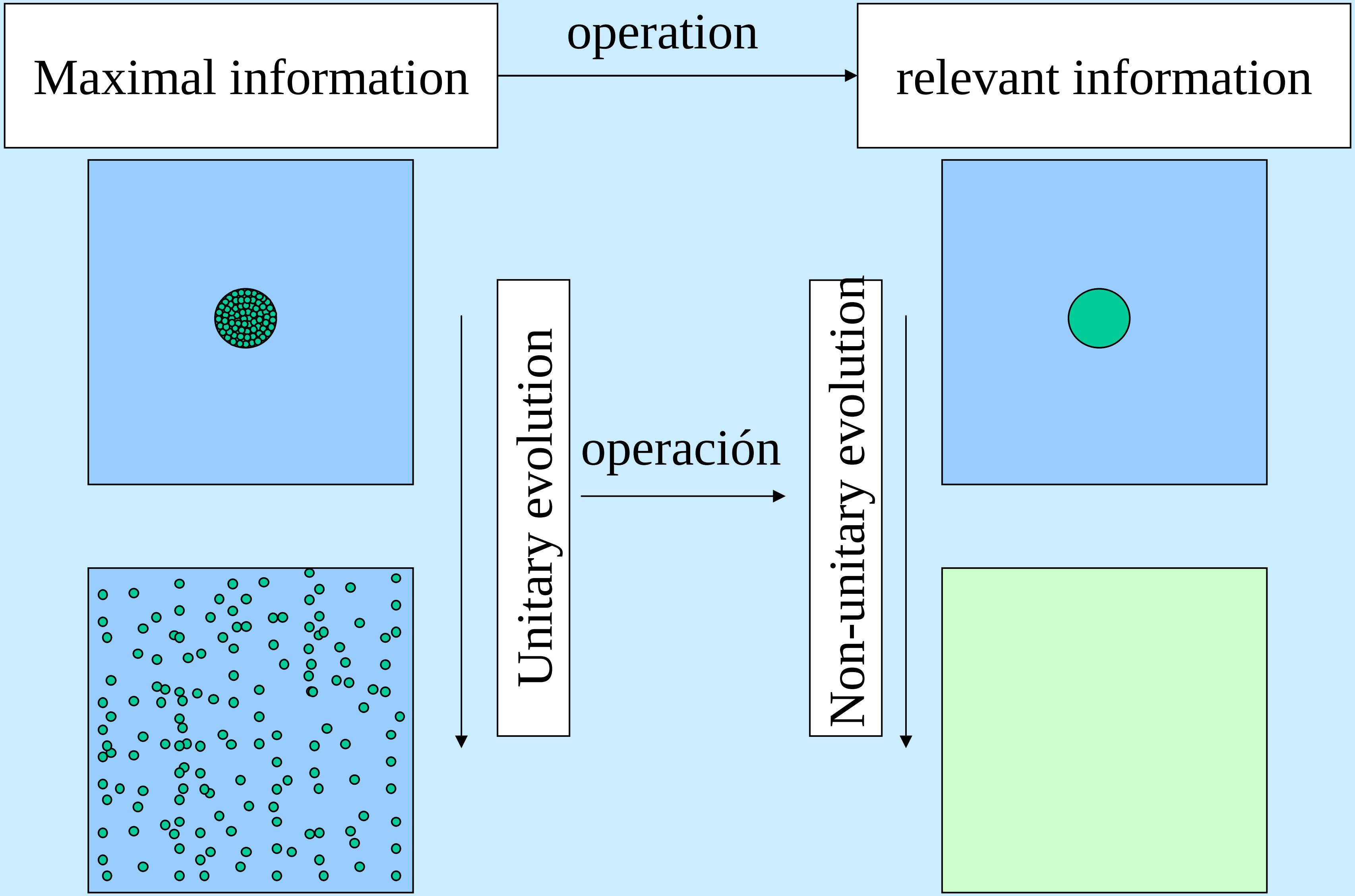
NON-UNITARY EVOLUTION



From a general point of view, this operation consists in the partitioning of the maximal information of the system in a relevant part and an irrelevant part.



NON-UNITARY EVOLUTION



In this kind of framework, the evolution would reach a situation of final equilibrium.

COARSE-GRAINING OPERATION

In QM the maximal information of a system U is given by the space *O* of all the observables that can be built for the system.

Maximal information $\longrightarrow$ $\mathcal{O}$

Relevant information $\longrightarrow$ $\mathcal{O}_R \subset \mathcal{O}$

Definition:

Coarse-grained state $\rho_G(t)$: $\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = \left\langle \hat{O}_R \right\rangle_{\hat{\rho}_G(t)}$

with $\hat{O}_R \in \mathcal{O}_R$

COARSE-GRAINING OPERATION

A coarse-graining operation is equivalent to a projection

It is known that:

$$\mathit{If}\ \mathcal{O}_R \subset \mathcal{O}\ \mathit{and}\ \hat{O}_R \in \mathcal{O}_R \Rightarrow \exists \hat{O} \in \mathcal{O} \,/\, \pi\hat{O} = \hat{O}_R\ ,\mathit{with}\ \pi\ \mathit{projector.}$$

It can be shown that ρ_G can be interpreted as the projection of ρ over $\mathcal{O}_R$'.

$$\mathit{If}\ \left\langle \hat{O}_R \right\rangle_{\hat{\rho}} = \left\langle \hat{O}_R \right\rangle_{\hat{\rho}_G} \Rightarrow \exists \pi \,/\, \hat{\rho}\pi = \hat{\rho}_G$$

COARSE-GRAINING OPERATION

Election of observables → Information loss

Unitary evolution → Non-unitary evolution

THE HAMILTONIAN OF THE CLOSED SYSTEM

Given the total Hamiltonian $H = H_0 + V$, where H_0 is the free Hamiltonian and V is a perturbation, V will be responsible for the introduction of poles. H_0 satisfies:

$$H_0|\omega\rangle = \omega|\omega\rangle \qquad \langle\omega|H_0 = \omega\langle\omega| \qquad 0 \le \omega \le \infty$$

and

$$I = \int_0^\infty d\omega |\omega\rangle\langle\omega|, \qquad \langle\omega|\omega'\rangle = \delta(\omega - \omega')$$

Then, $H_0 = \int_0^\infty \omega|\omega\rangle\langle\omega|$ and

$$H = H_0 + V = \int_0^\infty \omega|\omega\rangle\langle\omega| d\omega + \int_0^\infty d\omega \int_0^\infty d\omega' V_{\omega\omega'} |\omega\rangle\langle\omega'| = \int_0^\infty \omega|\omega^+\rangle\langle\omega^+| d\omega$$

THE EFFECTIVE *HAMILTONIAN*

Vectors $|\omega^+\rangle$ are the eigenvectors of, which are given by the Lippmann-Schwinger equations :

$$\langle\psi|\omega^+\rangle = \langle\psi|\omega\rangle + \langle\psi|\frac{1}{\omega + i0 - H}V|\omega\rangle$$

$$\langle\omega^+|\psi\rangle = \langle\omega|\psi\rangle + \langle\omega|\frac{1}{\omega + i0 - H}V|\psi\rangle$$

Poles
$z_n = \omega_n - i\gamma_n$

We assume that $\langle\psi|\omega\rangle$ and $\langle\omega|\psi\rangle$ are analytic functions in the whole complex plane. The second term in both equations introduces poles.
It is possible to build an effective Hamiltonian

$$H_{eff} = \sum_n z_n |z_n\rangle\langle z_n|$$

It can be shown that the poles appear in the mean values.

$$\langle O_R\rangle_{\rho_R(t)} = \langle O_R\rangle_{\rho_{Rdiag}{}^*} + \sum_n b_n(t)e^{-\gamma_n t} + Khalfin$$

TWO POLES WITHOUT KHALFIN

When we consider two poles and neglect the Khalfin term, we get:

$$\langle O_R \rangle_{\rho_R(t)} = \langle O_R \rangle_{\rho_{Rdiag}^{\ *}} + a_0(t)e^{-\gamma_0 t} + a_1(t)e^{-\gamma_1 t} \qquad \text{where} \qquad \gamma_0 << \gamma_1$$

So, we define the times of decoherence associated with γ_1 and relaxation with γ_0.

$$\boxed{t_D = \frac{1}{\gamma_1} \; and \; t_R = \frac{1}{\gamma_0}}$$

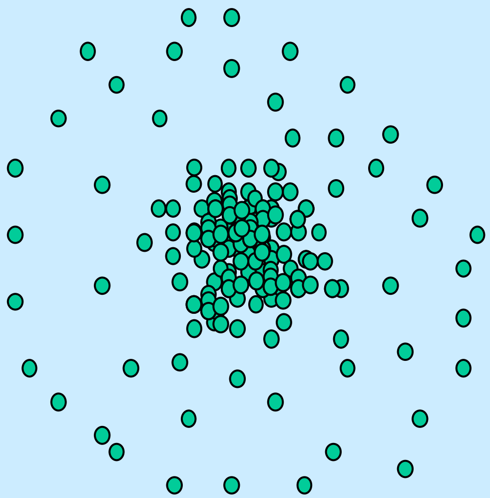
So after t_D

$$\langle O_R \rangle_{\rho_R(t)} \overset{t>t_D}{=} \langle O_R \rangle_{\rho_{Rdiag}^{\ *}} + a_0(t)e^{-\gamma_0 t} \qquad \to \sum_n \tilde{a}(t) |i(t)\rangle\langle i(t)|$$

The preferred basis is defined: is the one that banishes the interference.

Easy: With the poles, we determine t_D, then build the base is easy.

A GENERAL FRAMEWORK



1. Relevant observables are chosen

$$\hat{O}_R \in \mathcal{O}_R$$

2. Mean values are computed

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = Tr\left(\hat{O}_R \hat{\rho}(t)\right)$$

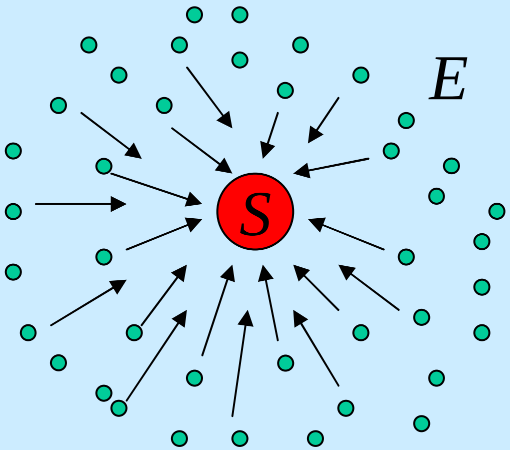
3. It can be demonstrated (when relaxation occurs) that mean values reach a final equilibrium value in a time t_R

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} \xrightarrow{t>t_R} \left\langle \hat{O}_R \right\rangle_{\hat{\rho}^*}$$

4. t_D and the preferred basis are computed by analizing the characteristic decay times of the mean values.

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = \Sigma^D(t) + \Sigma^{ND}(t) \xrightarrow[t\to t_D]{} \Sigma^D(t) \xrightarrow[t\to t_R]{} \Sigma^D(*)$$

THE FRAMEWORK INCLUDES EID



1. Relevant observables are chosen

$$\hat{O}_R = \hat{O}_S \otimes \hat{I}_E \in \mathcal{O}_R$$

2. Mean Values are calculated

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = Tr\left(\hat{O}_R \hat{\rho}(t)\right) = Tr\left(\hat{O}_S \hat{\rho}_S(t)\right) = \left\langle \hat{O}_S \right\rangle_{\hat{\rho}_S(t)}$$

3. It can be demonstrated (when there is relaxation)

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} \xrightarrow{t>t_R} \left\langle \hat{O}_R \right\rangle_{\hat{\rho}^*} \quad \equiv \quad \hat{\rho}_S(t) \xrightarrow{t>t_R} \hat{\rho}_S{}^*$$

4. It can be demonstrated (when there is decoherence)

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = \Sigma^D(t) + \Sigma^{ND}(t) \xrightarrow[t\to t_D]{} \Sigma^D(t) \xrightarrow[t\to t_R]{} \Sigma^D(*)$$

$$\equiv$$

$$\hat{\rho}_S(t) \xrightarrow[t\to t_D]{} \hat{\rho}_{S*}(t) \text{ diagonal} \xrightarrow[t\to t_R]{} \hat{\rho}_{S*}$$

NOTATION

It is convenient to use the following notation:

$$\mathcal{O}perators: |O)$$

$$\mathcal{S}tates: (\rho|$$

$$\mathcal{M}ean\ values: \langle O\rangle_\rho = (\rho|O)$$

- The operators belong to space $\mathcal{O}$.
- The states belong to space $\mathcal{O}$' (dual of O).
- The mean value is a real number.

APPLICATION TO CASES WITH NO ENVIRONMENT

Given a quantum system with Hamiltonian H with continuous spectrum:

$$H|\omega\rangle = \omega|\omega\rangle$$

1. We choose the *van Hove* observables $|O_R) \in \mathcal{O}_R$:

regular function

$$|O_R) = \int_0^\infty O(\omega)|\omega) d\omega + \int_0^\infty \int_0^\infty O(\omega,\omega')|\omega,\omega') d\omega d\omega'$$

The states ρ are represented by linear functionals on $\mathcal{O}'_R$

$$(\rho_R| = \int_0^\infty \rho(\omega)(\omega| d\omega + \int_0^\infty \int_0^\infty \rho(\omega,\omega')(\omega,\omega'| d\omega d\omega'$$

Cobasis

This restriction on the observables does not diminish the generality, because the observables not belonging to the van Hove space are not experimentally accessible

APPLICATION TO CASES WITH NO ENVIRONMENT

2. The expected value of an observable is:

$$\langle O_R \rangle_{\rho} = \int_0^{\infty} \rho^*(\omega) O(\omega) d\omega + \int_0^{\infty}\int_0^{\infty} \rho^*(\omega,\omega') O(\omega,\omega') d\omega d\omega'$$

The time evolution of this expected value is given by:

$$\langle O_R \rangle_{\rho(t)} = \int_0^{\infty} \rho^*(\omega) O(\omega) d\omega + \int_0^{\infty}\int_0^{\infty} \rho^*(\omega,\omega') O(\omega,\omega') e^{i\frac{\omega-\omega'}{\hbar}t} d\omega d\omega'$$

APPLICATION TO CASES WITH NO ENVIRONMENT

3. Since the functions are regular we can apply the Riemann-Lebesgue theorem, then:

$$\lim_{t\to\infty}\langle O_R\rangle_{\rho(t)} = \int_0^\infty \rho^*(\omega)O(\omega)d\omega$$

The mean value can be computed as if the system were in a stable final state:

$$\boxed{\lim_{t\to\infty}\langle O_R\rangle_{\rho(t)} = \langle O_R\rangle_{\rho_*}}$$

$$with \quad W-\lim_{t\to\infty}\rho(t) = \rho_* = \int_0^\infty \rho(\omega)(\omega| d\omega \; diagonal.$$

This means that the system decoheres on the basis of eigenvectors of the Hamiltonian.

APPLICATION TO CASES WITH NO ENVIRONMENT

1) The framework can be applied to SID (self-induced decoherence).

4. t_D and the preferred basis are computed by analizing the characteristic decay times of the mean values.

$$\langle \psi | \omega^+ \rangle = \langle \psi | \omega \rangle + \langle \psi | \frac{1}{\omega + i0 - H} V | \omega \rangle$$

$$\langle \omega^+ | \psi \rangle = \langle \omega | \psi \rangle + \langle \omega | \frac{1}{\omega + i0 - H} V | \psi \rangle$$

Poles
$z_n = \omega_n - i\gamma_n$

$$\left\langle \hat{O}_R \right\rangle_{\hat{\rho}(t)} = \Sigma^D(t) + \Sigma^{ND}(t) \xrightarrow[t \to t_D]{} \Sigma^D(t) \xrightarrow[t \to t_R]{} \Sigma^D(*)$$

CONCLUSIONS

In this talk we showed that:

➔ The ortodox approach of decoherence can not be applied to closed systems.

➔ The introduction of a coarse-graining operation transforms the unitary evolution into a non unitary one.

➔ The characteristic times of the system are given by the imaginary part of the poles of the Hamiltonian.

➔ The introduction of the polar technique to the *General Theoretical Framework for Decoherence* allows us to describe decoherence and relaxation in closed systems.

• M. Castagnino and S. Fortin, On a possible definition of the moving preferred basis, accepted in the proceedings of Quantum Physics with Non-Hermitian Operators 2011, Dresden. 2011.

• M. Castagnino and S. Fortin, New bases for a general definition for the moving preferred basis, *Mod. Phys. Lett. A* 26, 2365 (2011). arXiv:1103.6188

• M. Castagnino and S. Fortin, On a possible definition of the moving preferred basis, arXiv:1009.0535v2, 2010.

• M Castagnino et al, Class. Quantum Grav. 25 154002. 2008.

• M. Castagnino and R. Laura, Phys. Rev. A, 62, 022107, 2000.

• M. Castagnino and O. Lombardi, Phys. Rev. A, 72, 012102, 2005.

• R. Laura and M. Castagnino, Phys. Rev. A, 57, 4140, 1998.

• R. Laura and M. Castagnino, Phys. Rev. E, 57, 3948, 1998.