



# PT-symmetry in an optical fiber network

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**Max Planck Institute for the Science of Light, Erlangen, Germany**  
**CREOL, College of Optics and Photonics, University of Central Florida**



# MPL PT symmetry in an optical fiber network

Overview

## 1 Introduction / passive network

## 2 PT network with gain and loss

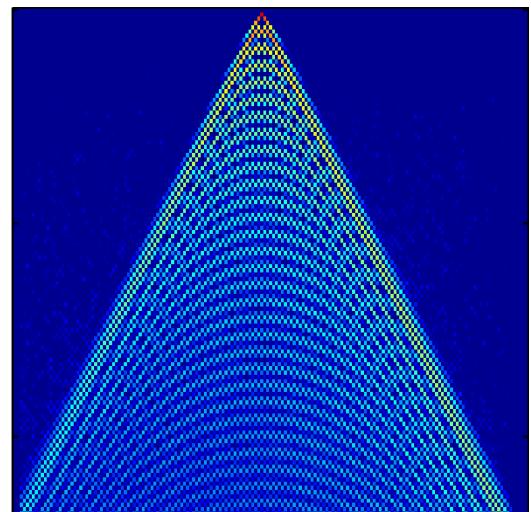
2.1 Realizing parity-time symmetry

2.2 Band structure

2.3 PT phase transition

2.4 PT scattering

## 3 Summary

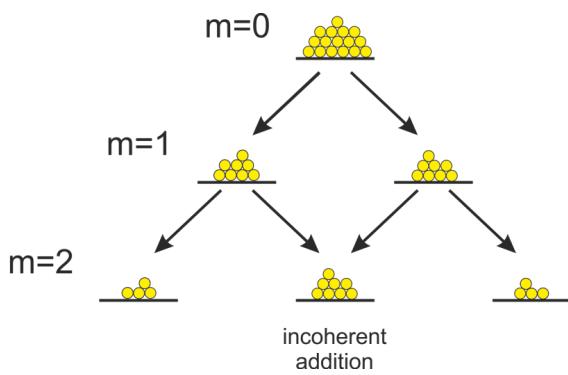


A. Regensburger et al, PRL 107, 233902 (2011)



## MPL Classical random walks vs. light walks

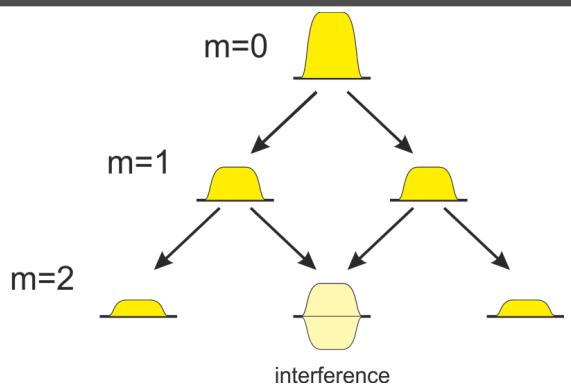
### Classical random walk



- Evolution in discrete steps  $m$
- Walker randomly steps to the left or right
- Incoherent addition (particle-like)

→ **Classical diffusion**

### Light walk



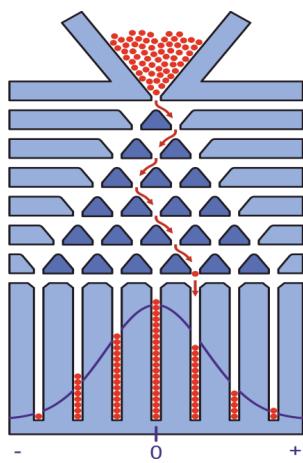
- Evolution in discrete steps  $m$
- Walker randomly steps to left or right
- Wave interference

→ **Ballistic spreading**



## MPL Implementation

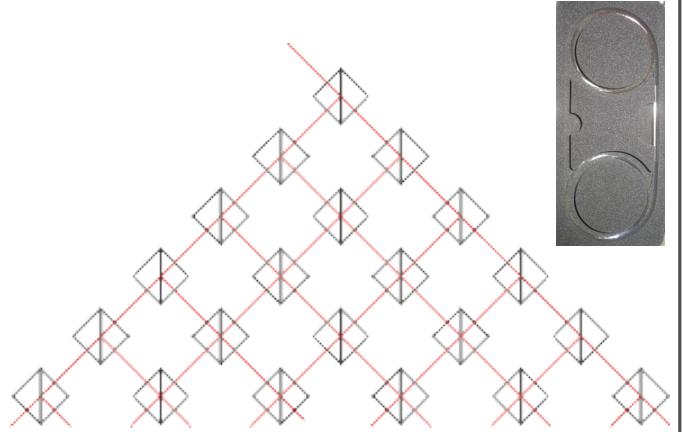
### Classical random walk



- Galton Board or „bean machine“



### Light walk



- Beamsplitter or coupler pyramid („Optical Galton Board“)
  - Too many components
  - Difficult to adjust

Source: Wikimedia Commons

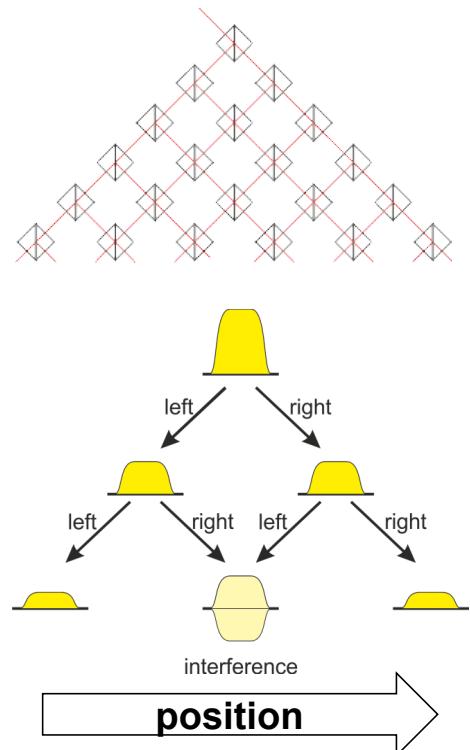
19.6.2012

D. Bouwmeester et al, Phys. Rev. A 61, 013410 (1999)

4

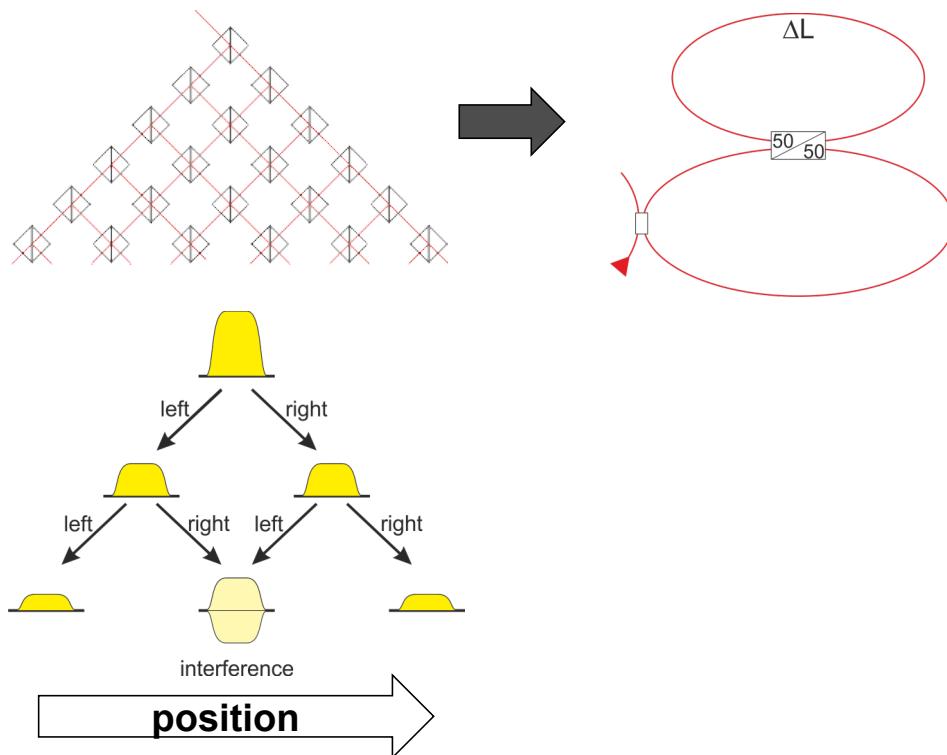


## MPL Solution: Time multiplexing





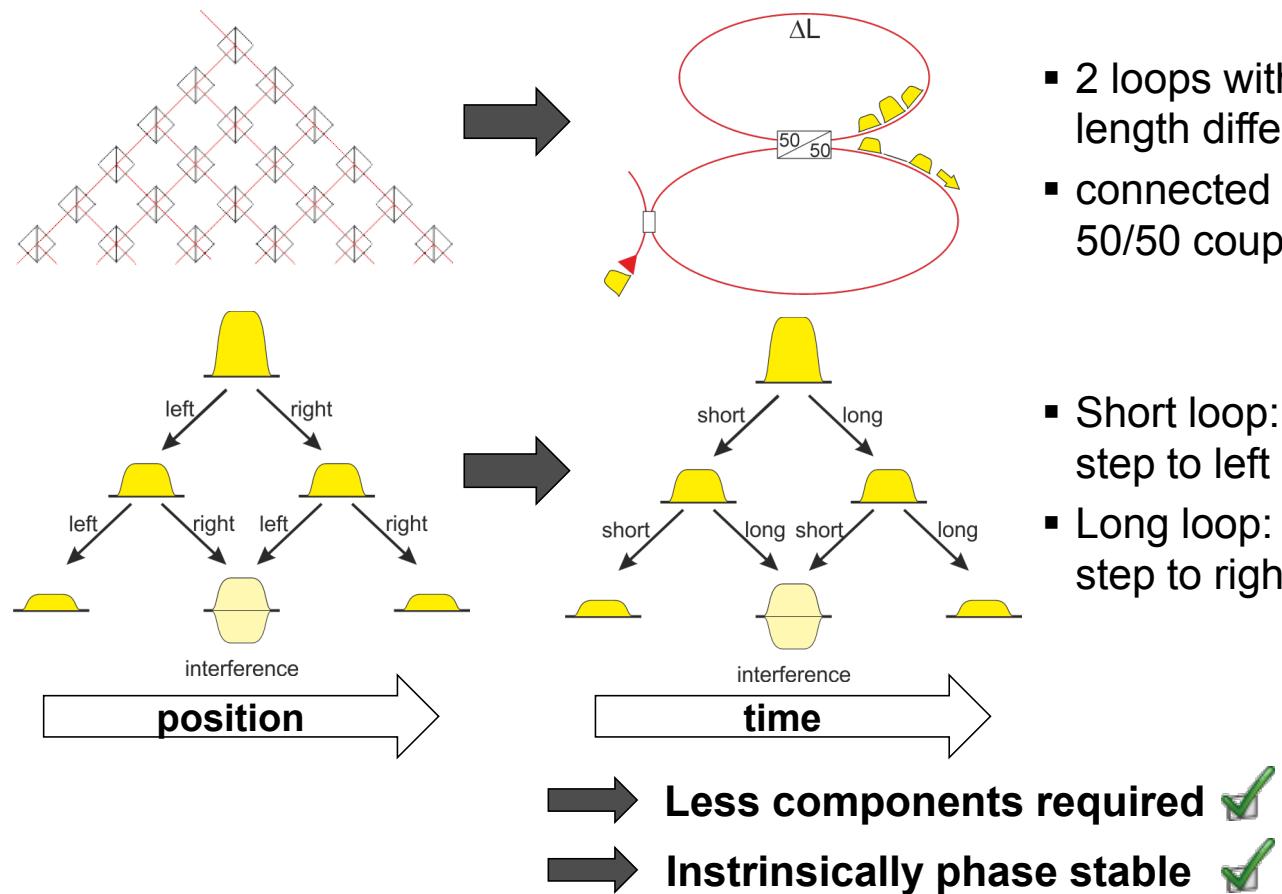
## MPL Solution: Time multiplexing



- 2 loops with length difference
- connected by 50/50 coupler

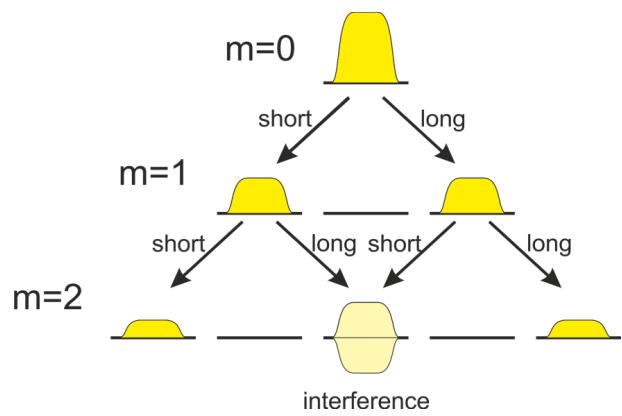
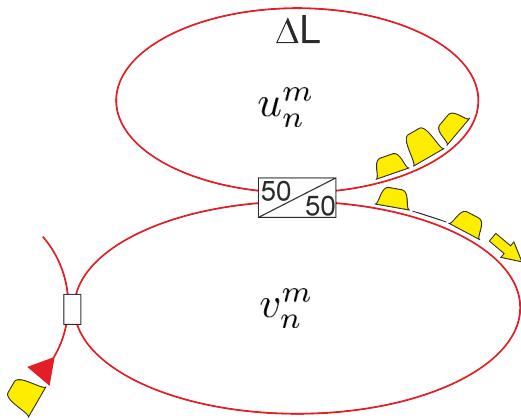


## MPL Solution: Time multiplexing





## MPL Dynamics of wave propagation



- Start: Single pulse at step (roundtrip)  $m=0$  and position  $n=0$
- Recursion equation:

$$\text{short loop: } u_n^{m+1} = \frac{1}{\sqrt{2}} (u_{n+1}^m + i v_{n+1}^m)$$

$$\text{long loop: } v_n^{m+1} = \frac{1}{\sqrt{2}} (i u_{n-1}^m + v_{n-1}^m) \exp(i\varphi(n))$$

$u_n^m$  : Amplitude in short loop

$v_n^m$  : Amplitude in long loop

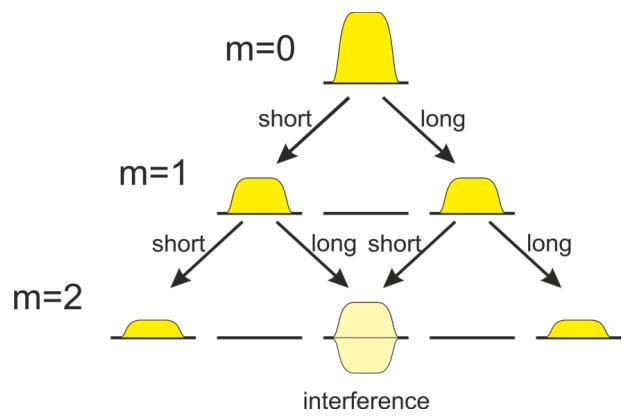
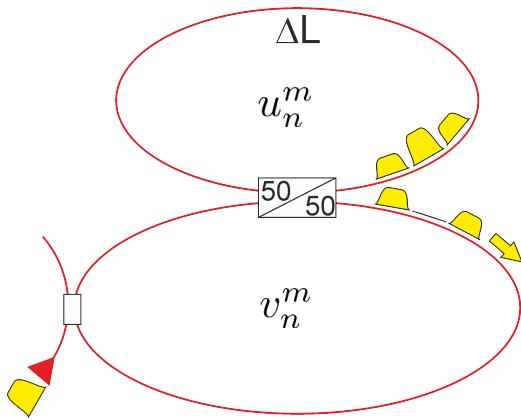
$n$  : Position on 1D grid

$m$  : Step (roundtrip)

$\varphi(n)$  : Phase potential



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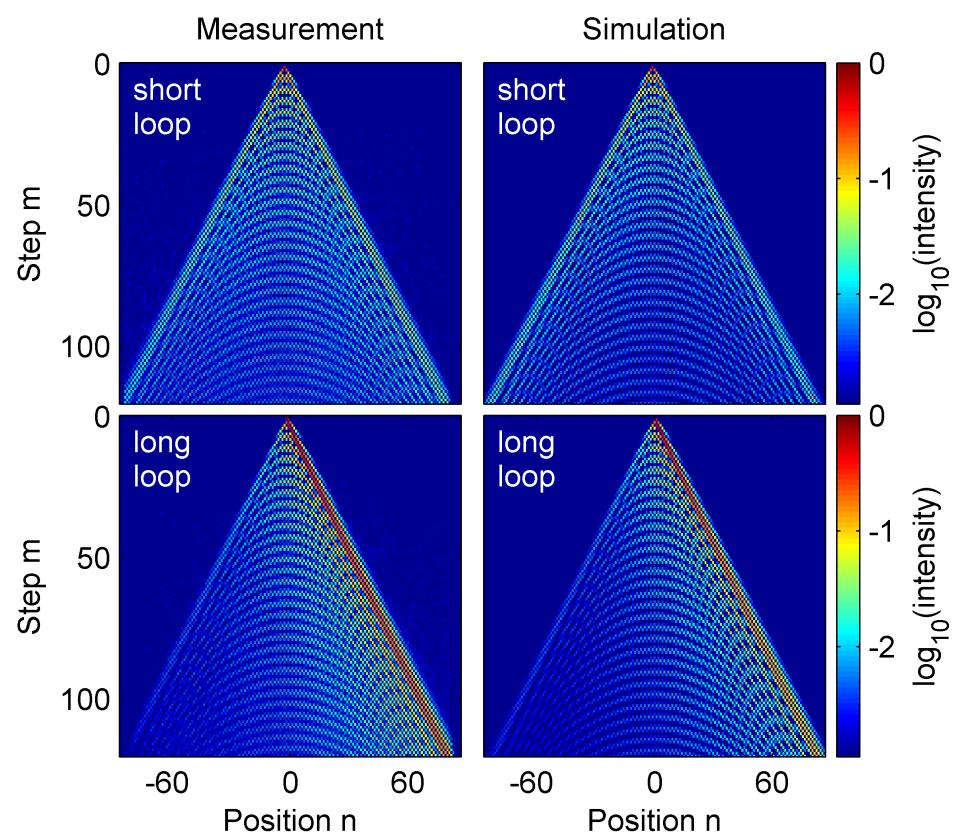
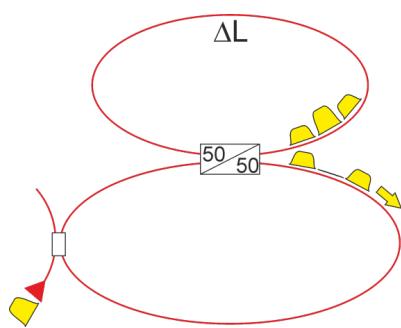
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## MPL Pulse spreading in the loops



→ Perfect agreement between experiment and simulation



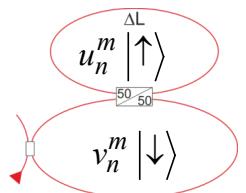
MPL

## Bandstructure of a light walk

### Recursion equations

$$\text{short loop: } u_n^{m+1} = \frac{1}{\sqrt{2}} (u_{n+1}^m + i v_{n+1}^m)$$

$$\text{long loop: } v_n^{m+1} = \frac{1}{\sqrt{2}} (i u_{n-1}^m + v_{n-1}^m)$$



### Bloch ansatz

$$u_n^m = U(\kappa) \exp [iQ n/4 + i\Theta m/2]$$

$$v_n^m = V(\kappa) \exp [iQ n/4 + i\Theta m/2]$$

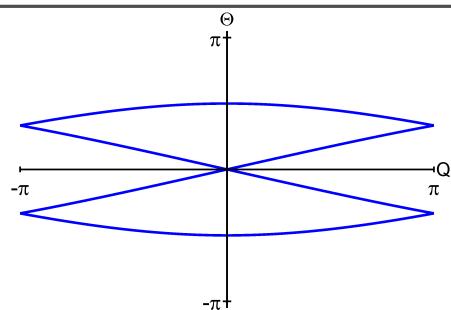
### Dispersion relation

$$\cos(Q) = 8 \cos^2(\Theta) - 8 \cos(\Theta) + 1$$

$\Theta$  : propagation constant

$Q$  : transverse wave number

### Band-structure



- 2 bands from 2 states (folded in)
- No central bandgap
- Real-valued

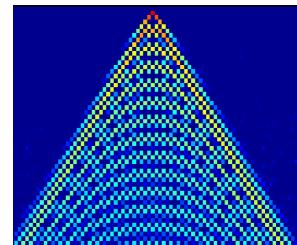
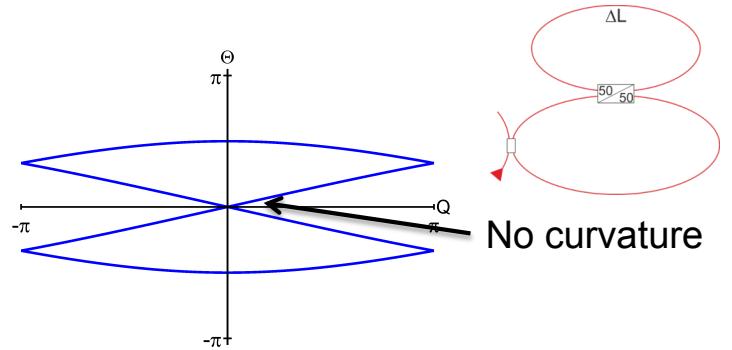
See reference: M.-A. Miri et al, Phys. Rev. A 86, 023807 (2012)



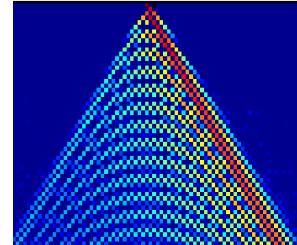
## MPL Bandstructure

- **Group velocity**  
 $\cong$  first derivative of bandstructure
- **Diffractive broadening**  
 $\cong$  second derivative / curvature

→ Regions with no curvature:  
Ballistic spreading  
with maximum velocity



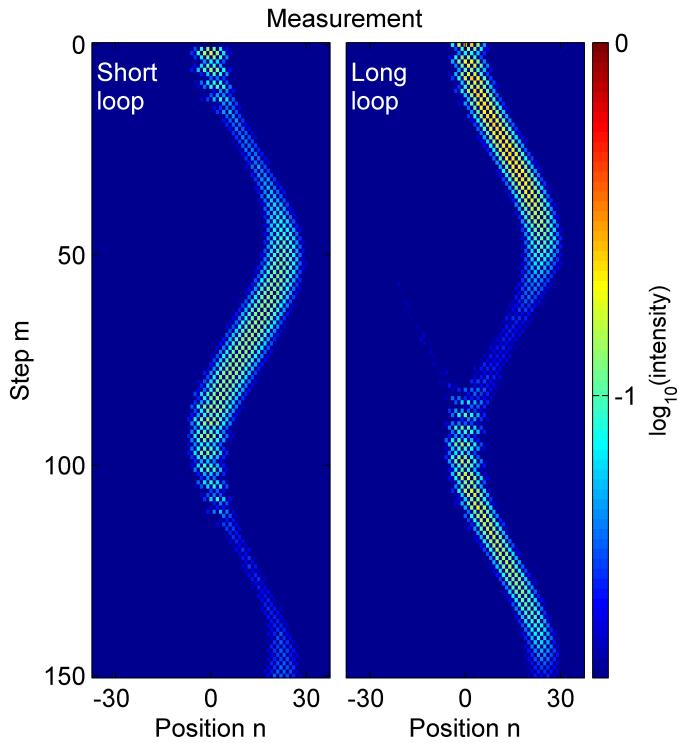
- **Interference between bands:**  
→ Zitterbewegung (trembling motion)  
Hyperbolic oscillations



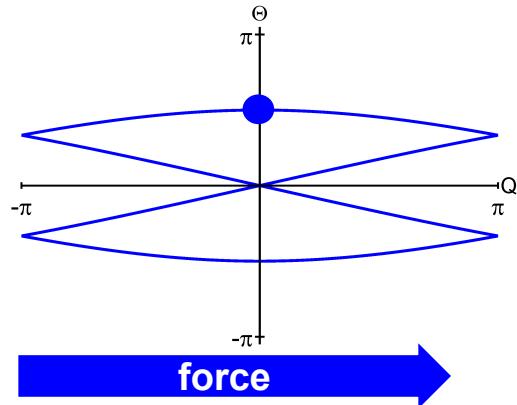


## MPL Bloch Oscillations

- Effect of energy or phase gradient:



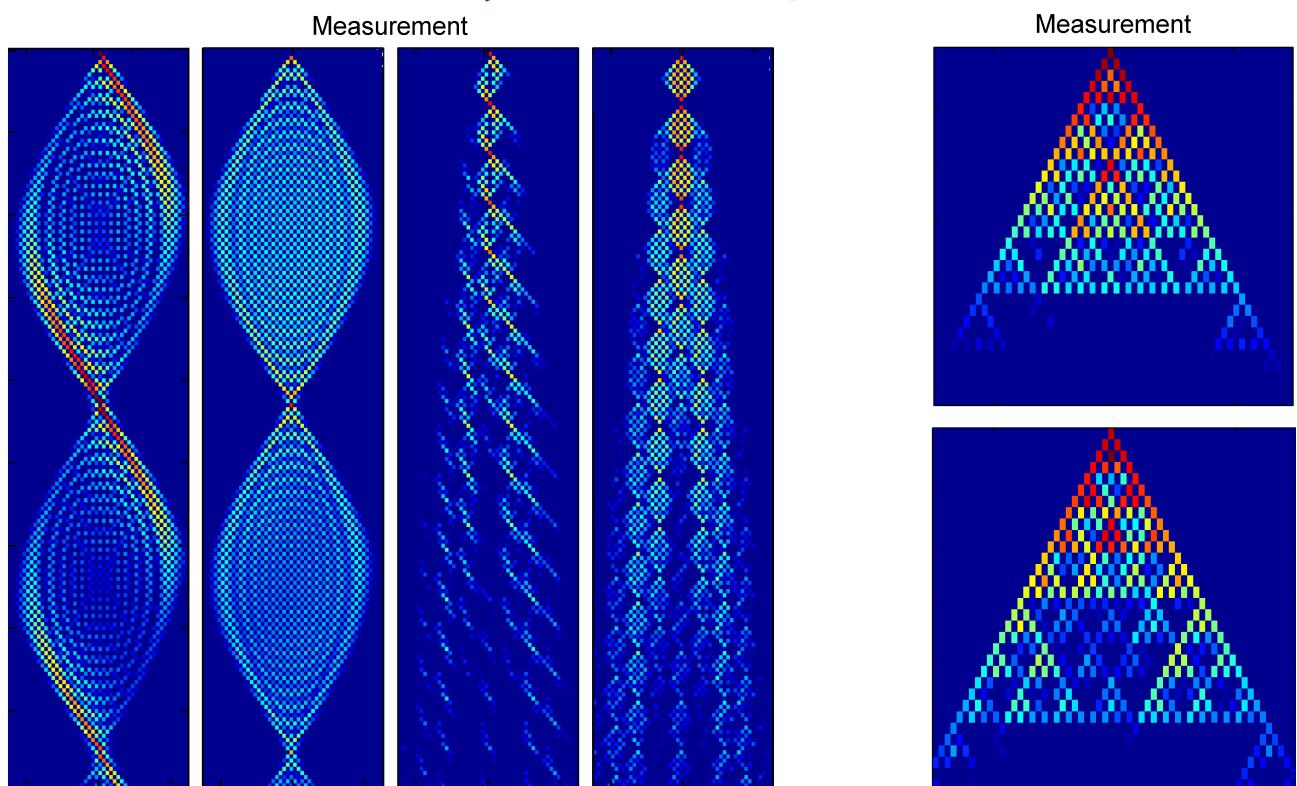
- Phase modulation
  - $v_n^m \rightarrow v_n^m \exp(-i\alpha n); \alpha \in [0; \pi]$
  - Acts as force on pulses
  - Periodicity: Return to initial state
- Oscillation



R.Morandotti, et al, Phys.Rev.Lett.83, 4756 (1999); T.Pertsch, et al, Phys.Rev.Lett.83, 4752 (1999); A. Wojcik et al, Phys. Rev. Lett. 93, 180601 (2004);



## MPL Various dynamics in passive fiber network



Bloch oscillations, Landau-Zener tunneling and fractal patterns

→ A. Regensburger et al, Phys. Rev. Lett. 107, 233902 (2011)



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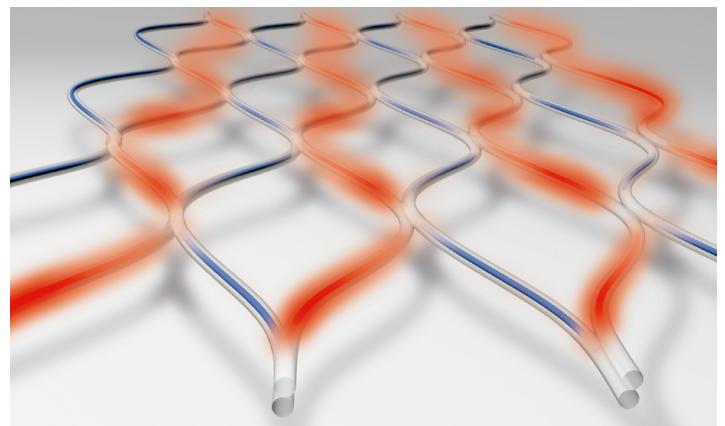
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2.1 Realizing parity-time symmetry

2.2 Band structure

2.3 PT phase transition

2.4 PT scattering



## 3 Summary

A. Regensburger et al, Nature 488, 167 (2012)



## MPL Introduction to PT symmetry

### Quantum mechanics

- Non-Hermitian Hamiltonians can have entirely real spectra, if they are PT-symmetric, i.e.

$$\mathcal{PT}\mathcal{H} = \mathcal{H}\mathcal{PT}$$

$\mathcal{P}$ : parity operator, i.e.  $\vec{x} \rightarrow -\vec{x}$

$\mathcal{T}$ : time reversal operator, i.e.  $t \rightarrow -t$

- Necessary:  $V(x) = V^*(-x)$

→  $\text{Re}[V(x)] = \text{Re}[V(-x)]$   
 $\text{Im}[V(x)] = -\text{Im}[V(-x)]$

- Symmetry suddenly broken above a certain PT threshold

→ complex eigenvalues

C. Bender et al, J. Math. Phys. 40, 2201 (1999)



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- Symmetry suddenly broken above a certain PT threshold

→ complex eigenvalues

## Analogy: Optical lattices

- Optical lattice with balanced gain and loss
- Refractive index  $n(x)$  plays role of  $V(x)$

$$n(x) = n_R(x) + i n_I(x)$$

gain/loss

- Necessary:  $n(x) = n^*(-x)$

→  $n_R(x) = n_R(-x)$   
 $n_I(x) = -n_I(-x)$

- Symmetry broken above critical gain/loss

→ complex propagation constants

- Experimentally realized in two-component device:

Rüter et al., Nature Phys.; Guo et al., PRL



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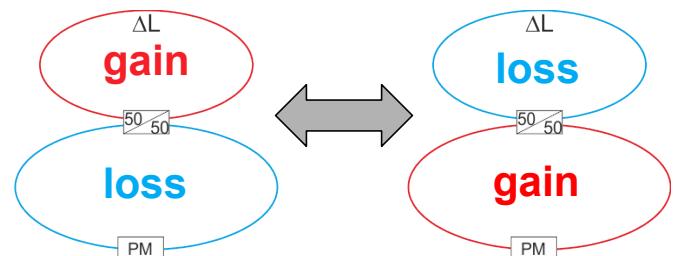
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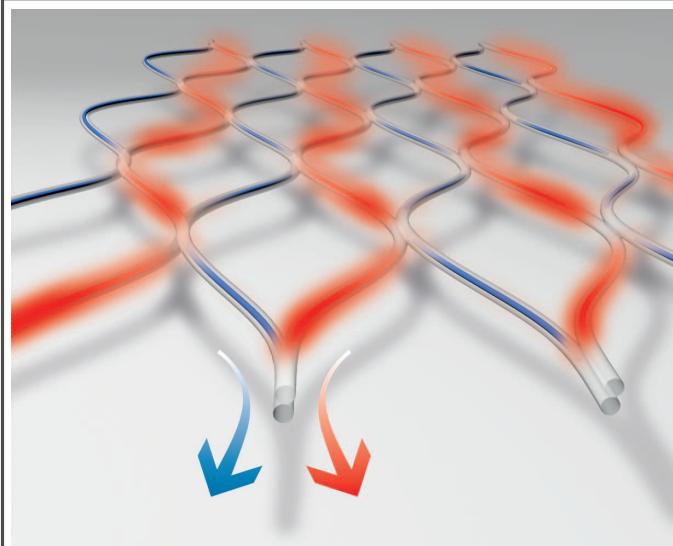


## 3 Summary



## MPL PT network

### PT fiber network



- Gain and loss fibers
- 50/50 couplers

### Explanation

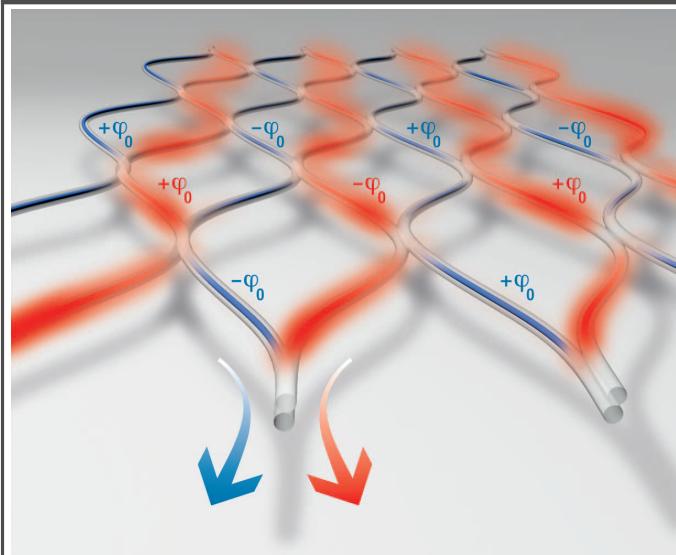
- Alternating gain and loss  
→  $n_I(x) = -n_I(-x)$  ✓
- No real part of refractive index  
→  $n_R(x) = 0$  ⚡
- **PT symmetry always broken (no threshold)**

A. Regensburger et al, Nature 488, 167 (2012)



## MPL Phase potential creates PT threshold

### PT fiber network



- Gain and loss fibers
- 50/50 couplers
- Phase modulation:  $\pm\varphi_0$

### Explanation

- Alternating gain and loss

$$\rightarrow n_I(x) = -n_I(-x) \quad \checkmark$$

- Symmetric phase potential



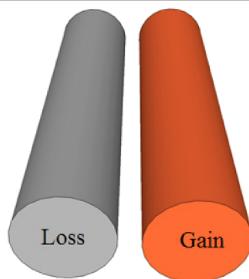
$$\rightarrow n_R(x) = n_R(-x) \quad \checkmark$$

→ Finite PT threshold



## MPL Why does the discrete work better?

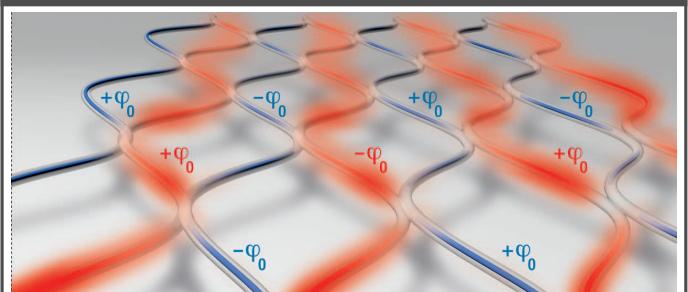
### PT waveguide array



- Continuous evolution
- Applied all at the same time:
  - Gain / loss
  - Phase potential
  - Coupling

→ Further complicated by Kramers-Kronig relations

### PT fiber network



- Discrete in evolution
- Completely independent:
  - Gain/loss
  - Phase potential
  - Coupling

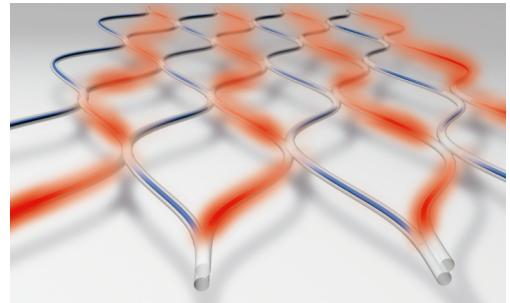
→ Full flexibility, can be transferred to time domain



## MPL Experimental realization with loops

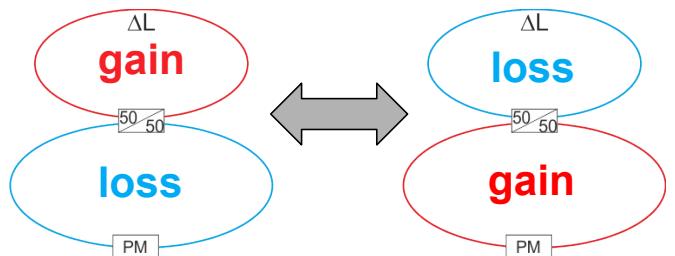
Fiber array

- Real space picture



Time multiplexing

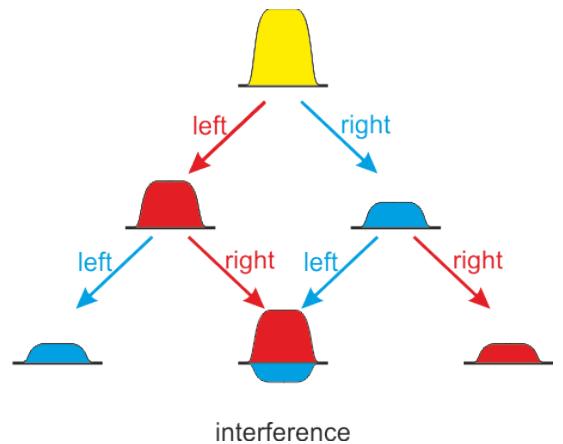
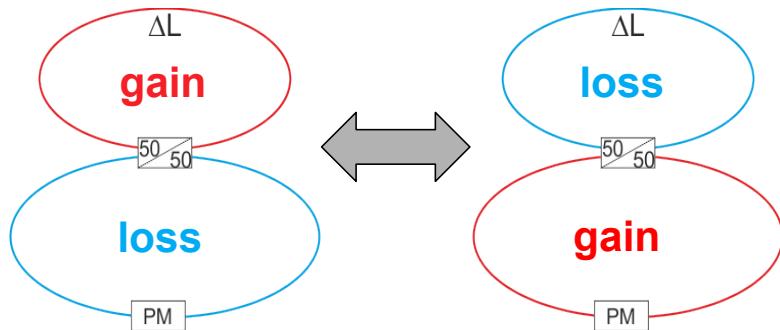
- Emulated by two-loop setup
- Gain/Loss switched between loops in every roundtrip
- Potential applied by phase modulator



A. Regensburger et al, Nature 488, 167 (2012)



## MPL PT Dynamics of wave propagation



**Every odd step:**

$$\text{short loop: } u_n^{m+1} = \frac{\sqrt{G}}{\sqrt{2}} (u_{n+1}^m + i v_{n+1}^m)$$

$$\text{long loop: } v_n^{m+1} = \frac{1}{\sqrt{G}\sqrt{2}} (i u_{n-1}^m + v_{n-1}^m) \exp(i\varphi(n))$$

**Every even step:**

$$\text{short loop: } u_n^{m+1} = \frac{1}{\sqrt{G}\sqrt{2}} (u_{n+1}^m + i v_{n+1}^m)$$

$$\text{long loop: } v_n^{m+1} = \frac{\sqrt{G}}{\sqrt{2}} (i u_{n-1}^m + v_{n-1}^m) \exp(i\varphi(n))$$

$u_n^m$  : Amplitude in short loop

$v_n^m$  : Amplitude in long loop

$n$  : Position on 1D grid

$m$  : Step (roundtrip)

$\varphi(n)$  : Phase potential function

$G$  : Gain or 1/loss

A. Regensburger et al, Nature 488, 167 (2012)



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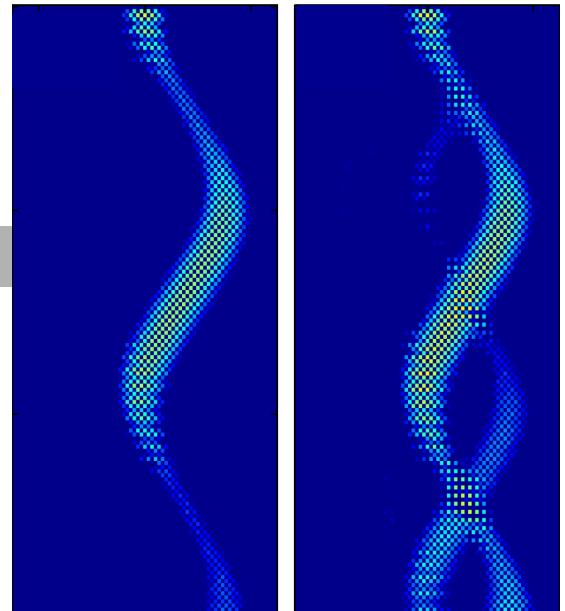
2.1 Realizing parity-time symmetry

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2.4 PT scattering

## 3 Summary





## MPL PT band structure

### Dispersion relation

- **Passive:**

$$\cos(Q) = 8 \cos^2(\Theta) - 8 \cos(\Theta) + 1$$

- **Gain, loss and phase potential:**

$$\begin{aligned} \cos(Q) = & 8 \cos^2(\Theta) - 8 \cosh(\gamma) \cos(\varphi_0) \cos(\Theta) \\ & + 4 \cos^2(\varphi_0) - 4 + \cosh(2\gamma) \end{aligned}$$

$\Theta$  : propagation constant

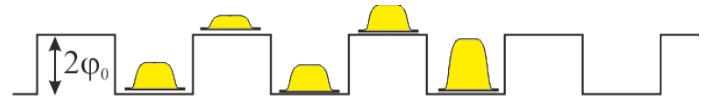
$Q$  : transverse wave number

$\varphi_0$  : phase potential

$\gamma = \log(G)$  : gain/loss

### Shape of phase potential:

$$\varphi(n) = \begin{cases} -\varphi_0 & \text{for } \mod(n+3; 4) = 0 \text{ or } 1 \\ +\varphi_0 & \text{for } \mod(n+3; 4) = 2 \text{ or } 3 \end{cases}$$





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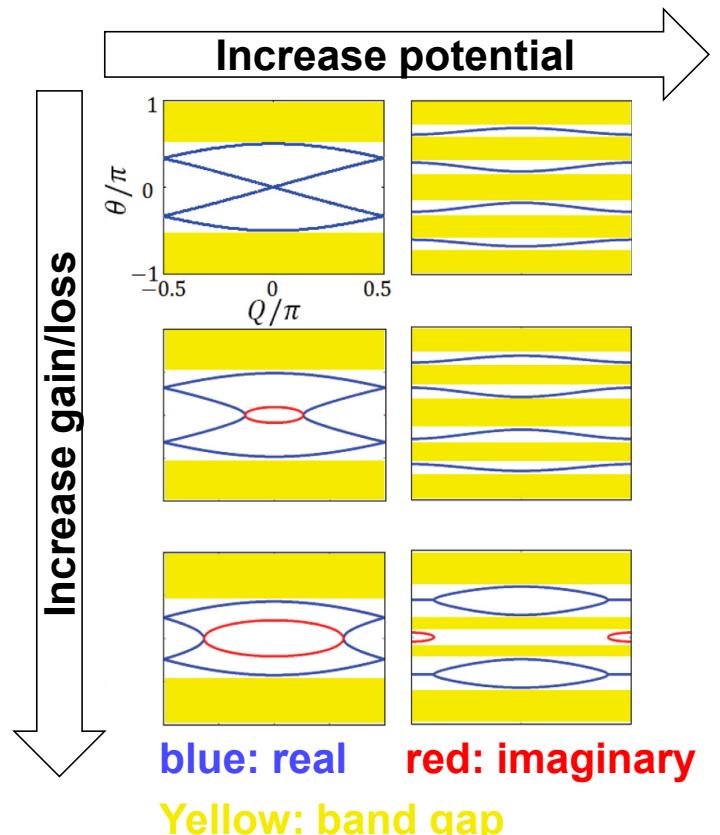
$Q$  : transverse wave number

$\varphi_0$  : phase potential

$\gamma = \log(G)$  : gain/loss

→ Potential pushes bands apart

→ Gain/loss lead to band merging  
(complex eigenvalues)

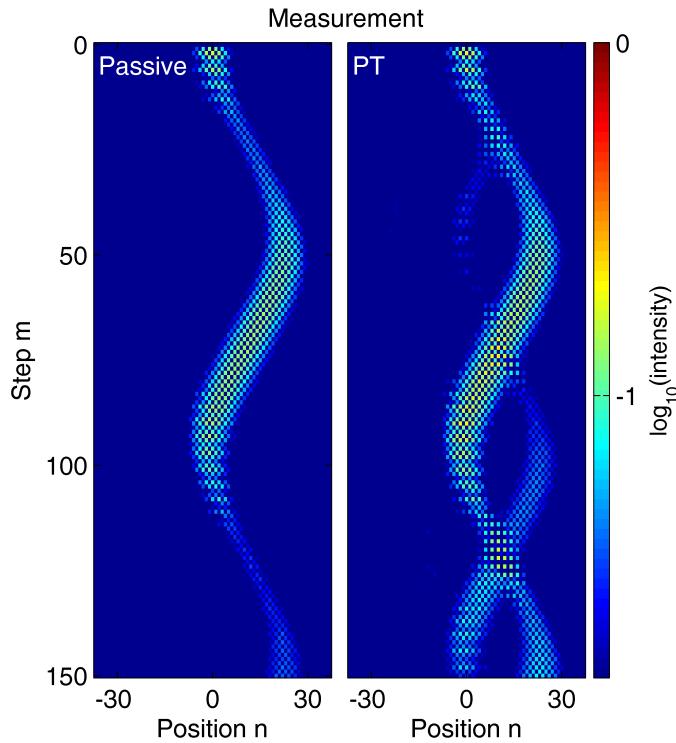


M.-A. Miri et al, Phys. Rev. A 86, 023807 (2012)

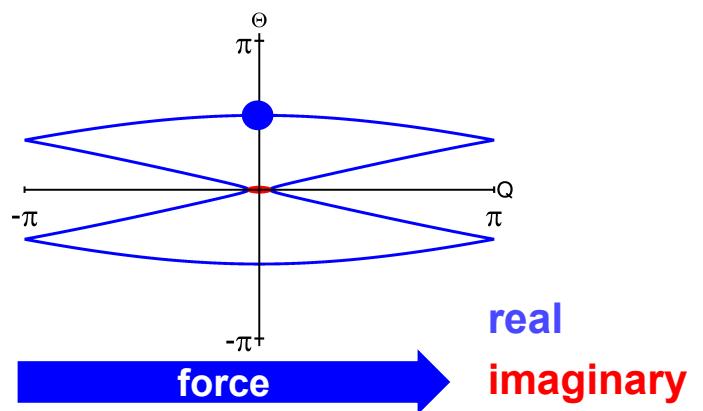


## MPL PT Bloch oscillations

- Slight amount of gain/loss



- Phase modulation
  - $v_n^m \rightarrow v_n^m \exp(-i\alpha n); \alpha \in [0; \pi]$
  - Scans PT band structure
  - Imaginary eigenmodes
- periodic emission of radiation



S. Longhi et al, Phys. Rev. Lett. 103, 123601 (2009)



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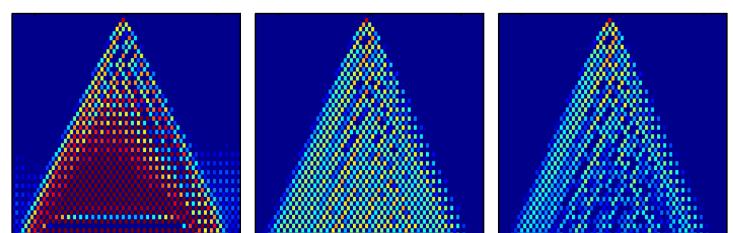
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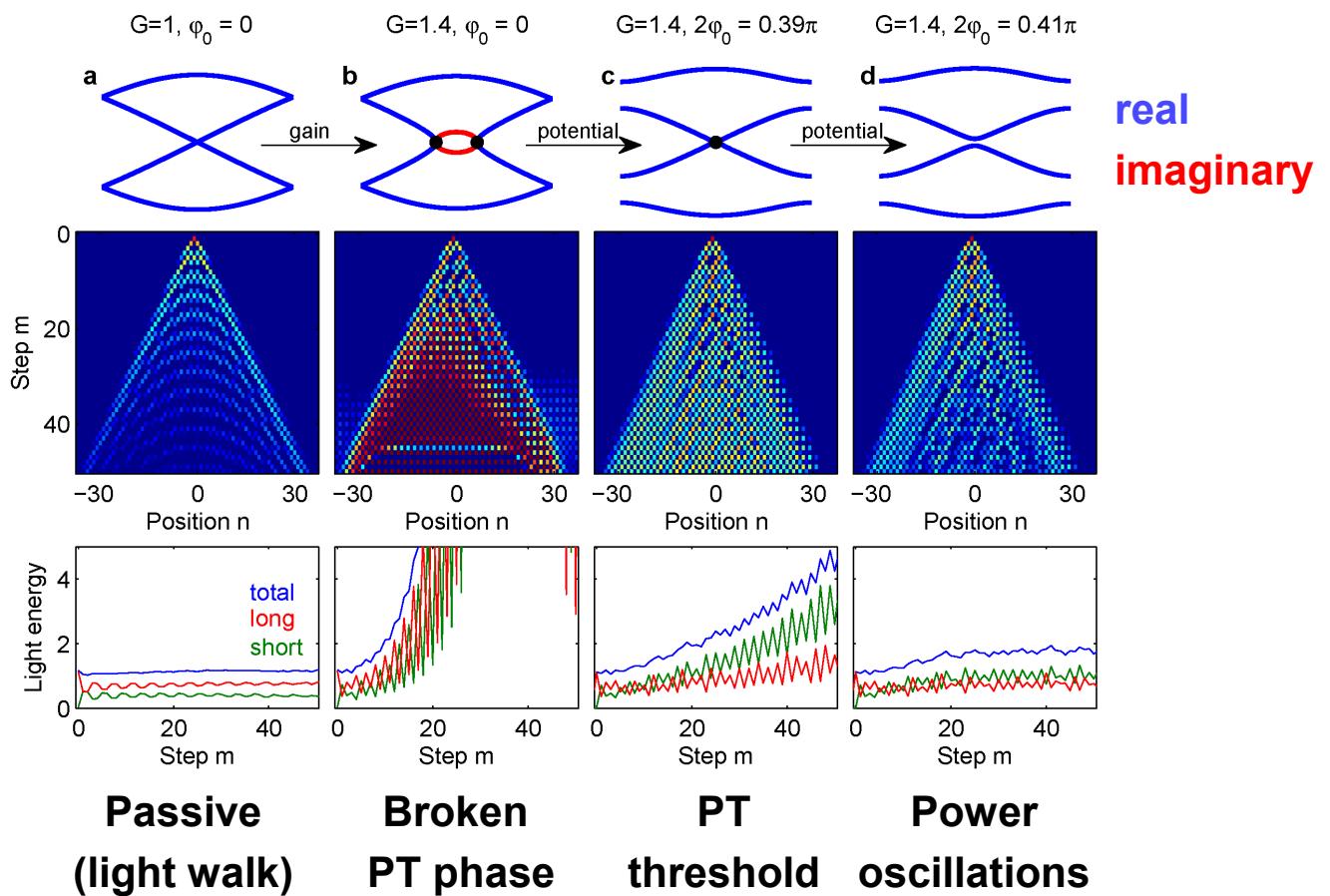
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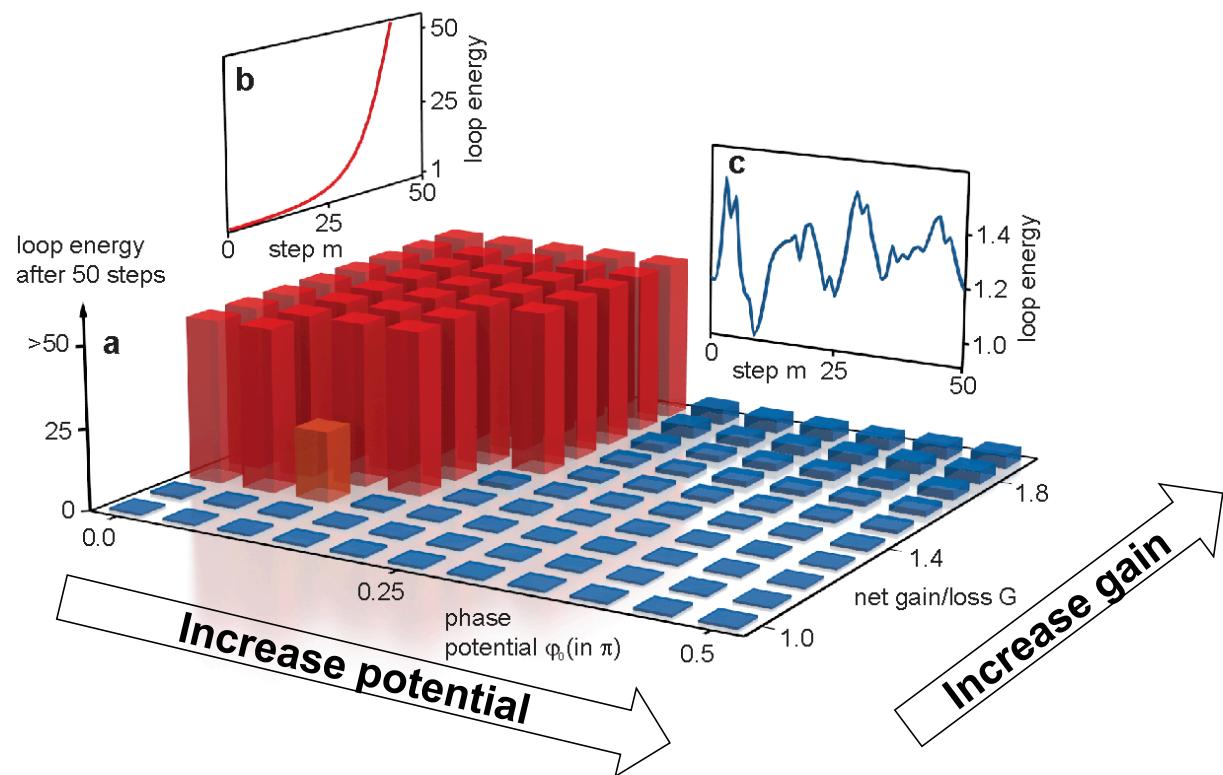


## MPL PT phase transition (measurement)





## MPL Parameter scan (measured)



→ Transition from growing to almost stable modes explored

A. Regensburger et al, Nature 488, 167 (2012)



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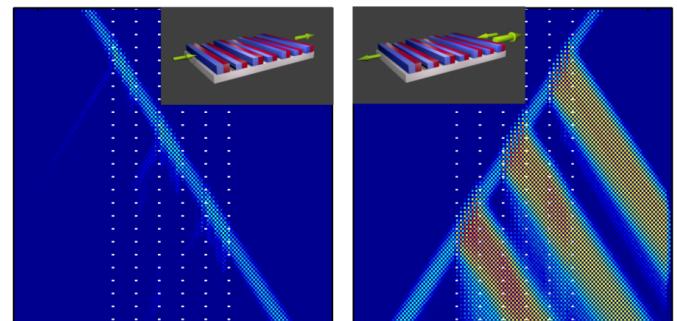
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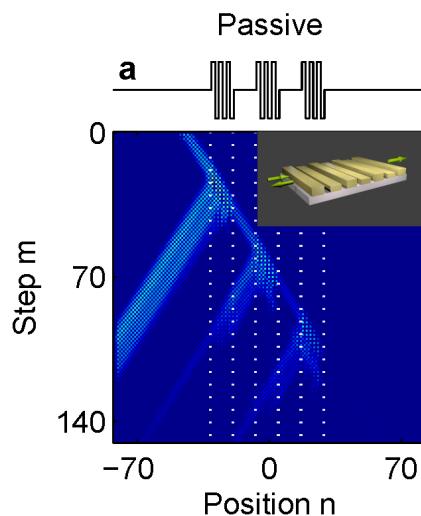
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## MPL Scattering (measured)

### Passive

- Temporal Bragg scatterers by phase modulation
- Incoming pulse is reflected:



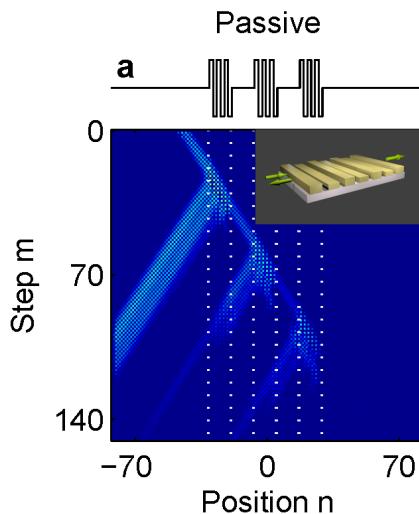
Z. Lin et al, Phys. Rev. Lett. 106, 213901 (2011); A. Regensburger et al, Nature 488, 167 (2012)



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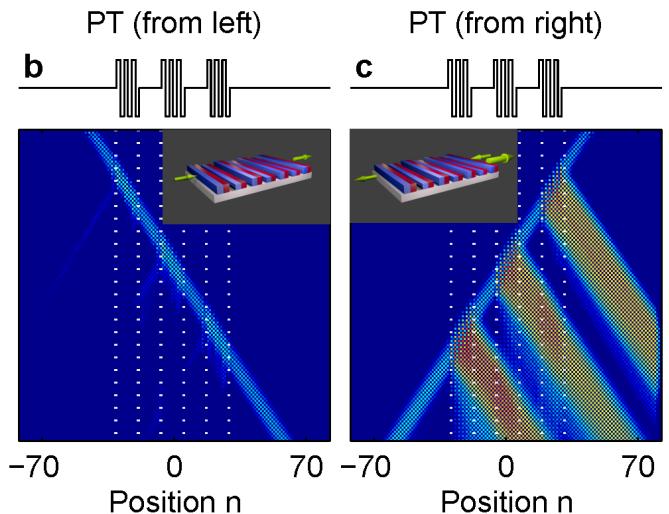
### Passive

- Temporal Bragg scatterers by phase modulation
- Incoming pulse is reflected:



### PT: Unidirectional Invisibility

- Add gain and loss to scatterers (exactly at PT threshold)
- Unidirectional invisibility:



Z. Lin et al, Phys. Rev. Lett. 106, 213901 (2011); A. Regensburger et al, Nature 488, 167 (2012)



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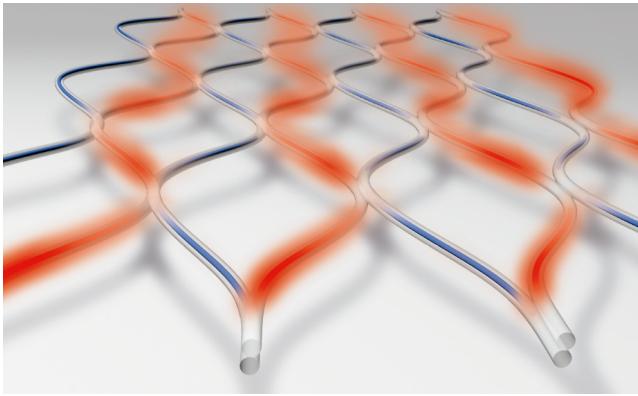
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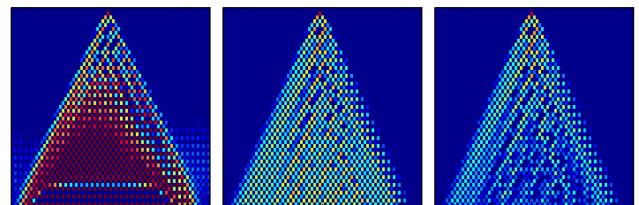
### Concept

- PT-symmetric distribution of **gain**, **loss** and phase potential in fiber loops
- Equivalent spatial network:

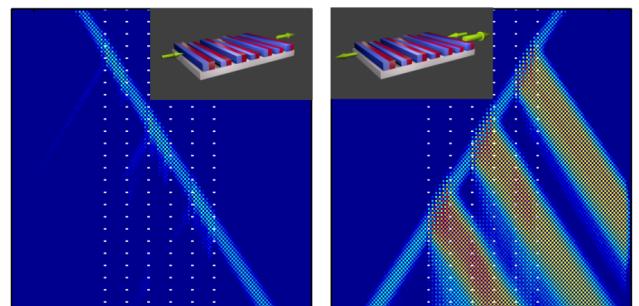


### Experiments

- PT phase transition:



- Unidirectional invisibility:



References: A. Regensburger et al, Nature 488, 167 (2012) & M.-A. Miri et al, Phys. Rev. A 86, 023807 (2012)



## MPL Open questions

### Questions

#### Discreteness

- Differences to continuous systems?
- New possibilities?

#### Invisibility

- Why is this possible in a discrete system?

#### Applications

- Consequences for communication systems?

