Landau-Zener-Stueckelberg interferometry in PT-symmetric optical waveguides

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Outline

- Motivation:
  - PT-symmetry
  - Optical waveguides
- The Model
  - Forced Schrödinger equation
  - Effective two level system
  - Driven two level system
- Landau-Zener-Stückelberg
  - Non-symmetric Landau-Zener-Stückelberg interferometry
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- Conclusions

PT-symmetric hamiltonians

- Conventionally we require Hermiticity to ensure that the Hamiltonian has real spectrum and unitary evolution.

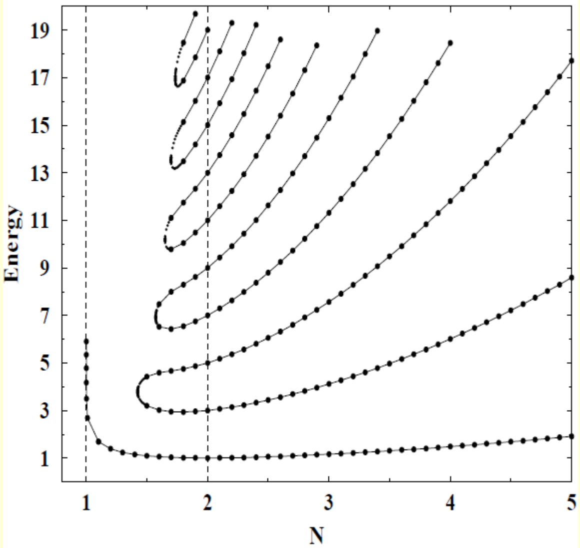
- If we require instead PT-symmetry invariance, we can also have real spectrum and unitary evolution.

$$\boxed{\begin{array}{c} \mathcal{P}: x \to -x, \quad p \to -p \\ \mathcal{T}: x \to x, \quad p \to -p, \quad i \to -i \end{array}}$$

For example,

$$\boxed{H = p^2 - (ix)^N}$$

$$N_{crit} = 2$$



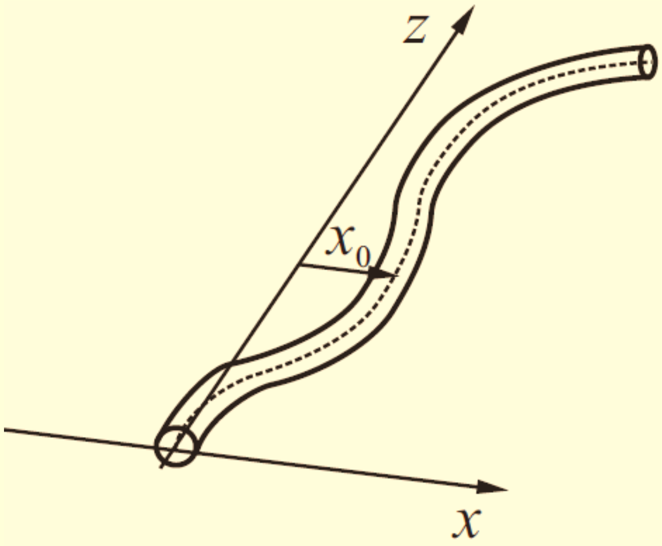
Bender and Boettcher, PRL (1998)
Bender, Rep. Prog. Phys. (2007)

Optical waveguides

In the scalar and paraxial approximation the equations for wave propagation look exactly like the Schrödinger equation.

$$\boxed{i\lambdabar\partial_Z\psi = -\frac{\lambdabar^2}{2n_s}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi - F(Z)x\psi \equiv \mathcal{H}_0\psi - F(Z)x\psi}$$

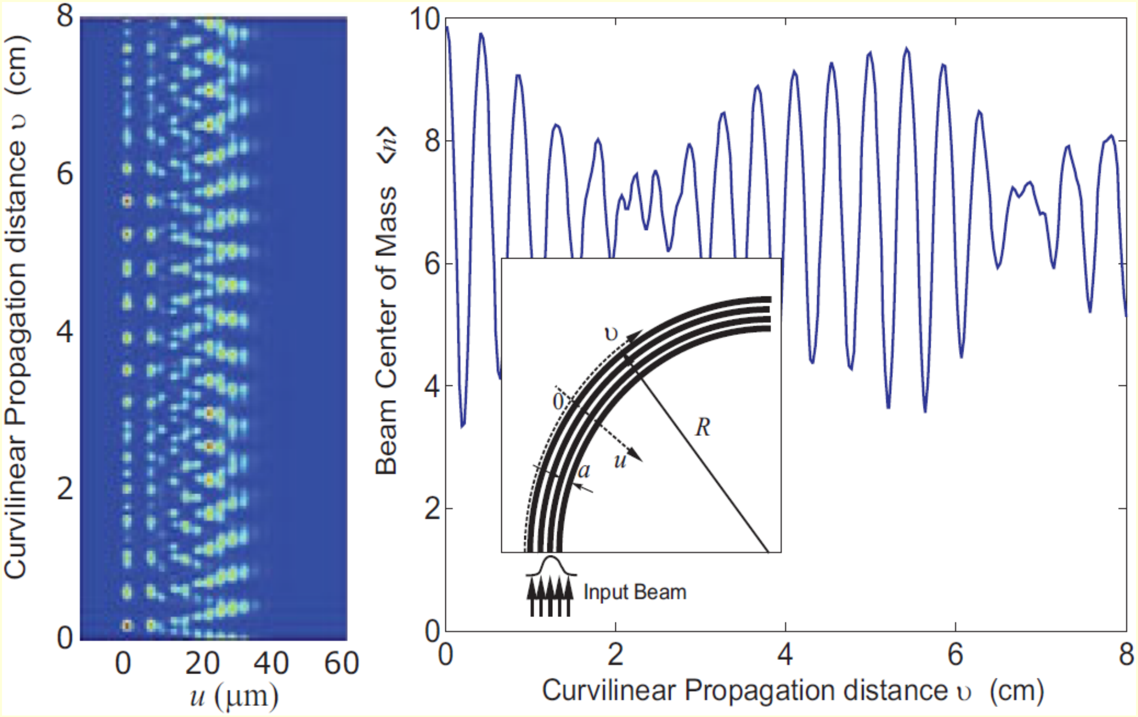
$$\lambdabar = \lambda/2\pi \qquad x = X - X_0(Z)$$



$$\boxed{\begin{aligned} U(x) &\approx n_s - n(x) \\ F(Z) &= -n_s\ddot{X}_0(Z) \end{aligned}}$$

Optical waveguides

Engineered photonic waveguides can be used as a classical analog of QM.



S. Longhi, Laser & Photon. Rev. (2009)

For example, a quantum particle bouncing under the effect of gravity can be visualized on a curved array of waveguides.

LETTERS
PUBLISHED ONLINE: 24 JANUARY 2010 | DOI: 10.1038/NPHYS1515

nature physics

Observation of parity–time symmetry in optics

Christian E. Rüter[1], Konstantinos G. Makris[2], Ramy El-Ganainy[2], Demetrios N. Christodoulides[2], Mordechai Segev[3] and Detlef Kip[1]*

ARTICLE

doi:10.1038/nature11298

Parity–time synthetic photonic lattices

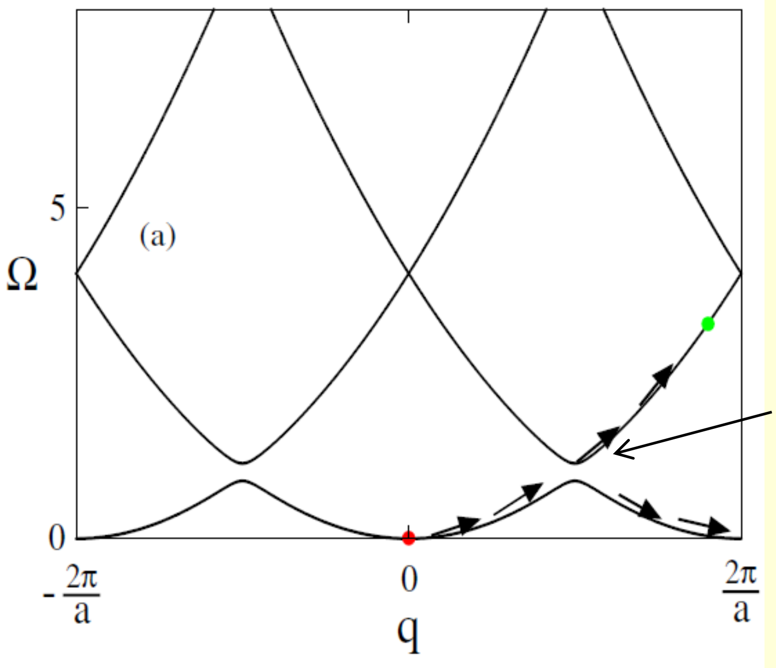
Alois Regensburger[1,2], Christoph Bersch[1,2], Mohammad-Ali Miri[3], Georgy Onishchukov[2], Demetrios N. Christodoulides[3] & Ulf Peschel[1]

PT-symmetric lattice

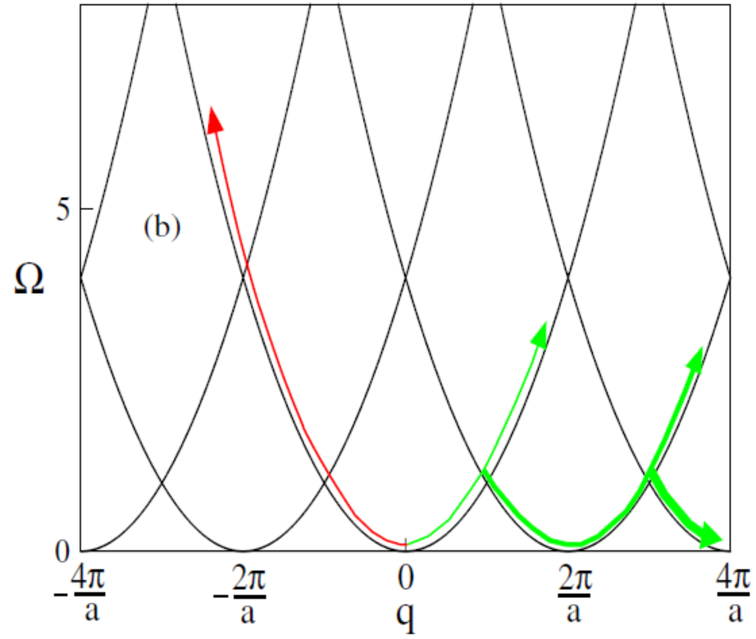
By appropiately tayloring the gain and loss regions a *PT*-symmetric lattice may be realized.

$$U(x) \;=\; U_1 \cos(2\pi x/a) + iU_2 \sin(2\pi x/a)$$

K. Makris et. al. , PRL (2008) and PRA (2010)
S. Longhi, PRL (2009)



$F(Z) = F$

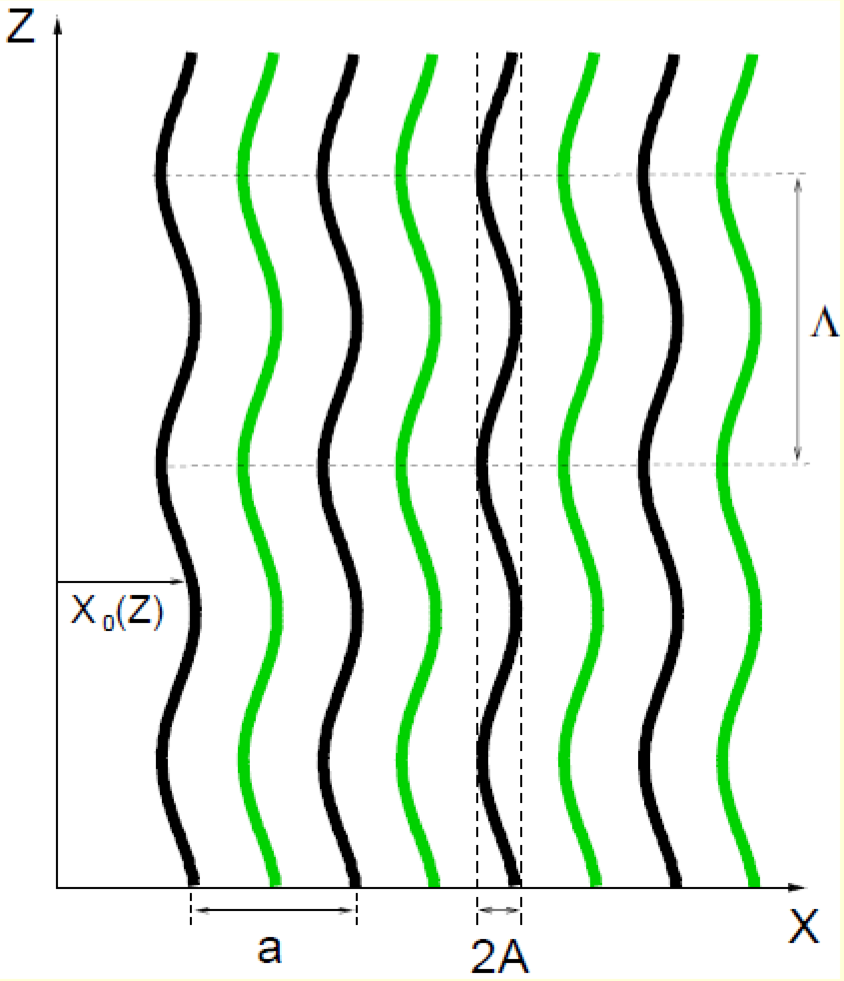


Morales-Molina and SR, J. Phys. B. (2011)

Eigenvalues of H_0 are real for: $U_2 \;<\; U_2^{crit} \;=\; U_1$

PT-symmetric lattice

A sinusoidal curvature along the propagation direction mimics an AC field.



$$X_0(Z) = A\cos(2\pi Z/\Lambda)$$

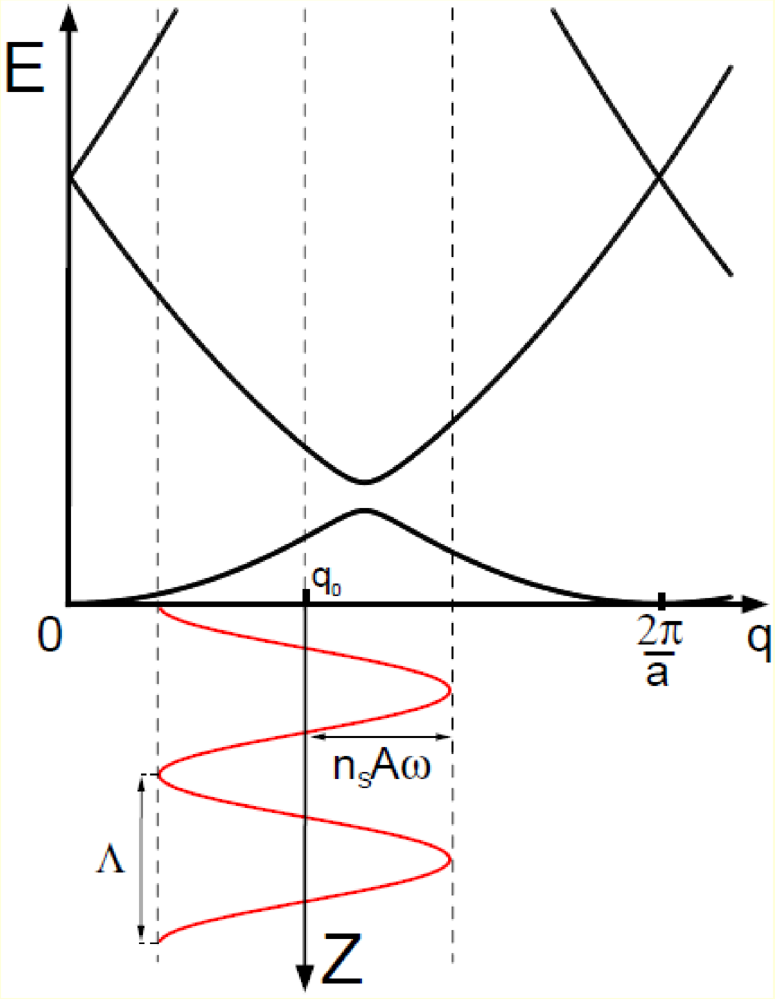
$$\Downarrow$$

$$\boxed{F(Z) = n_s A\omega^2 \cos(\omega Z)}$$

$$\omega = 2\pi/\Lambda$$

(Alternate driving "field")

Driving around a level "crossing"

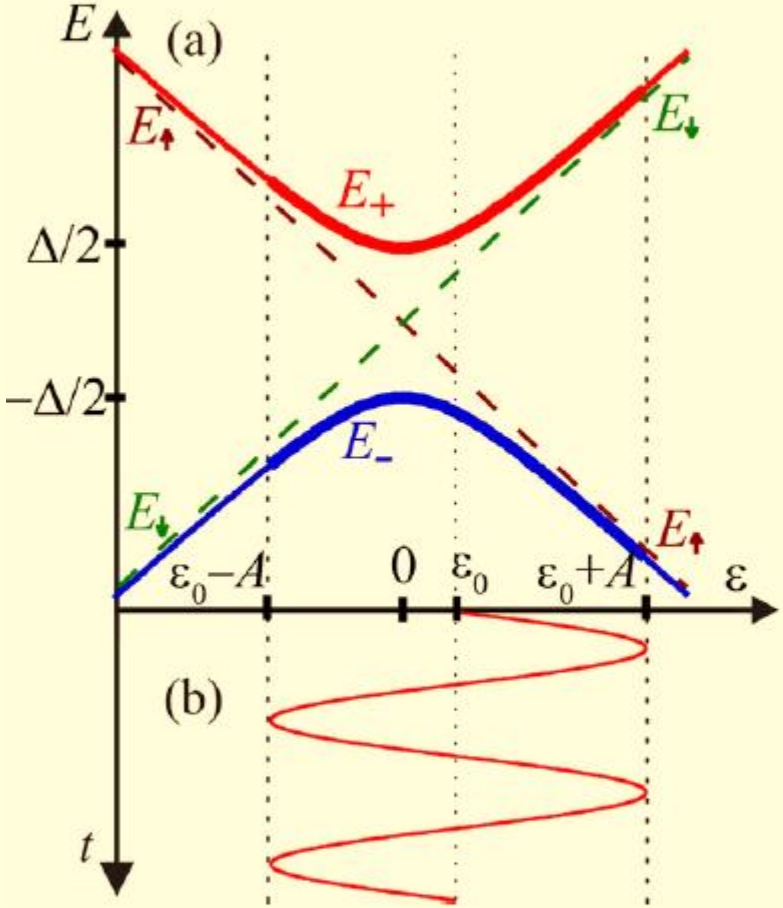


$$\boxed{q(Z) = q_0 + n_s A\omega \sin(\omega Z)}$$

Evolution of the quasimomentum under the AC driving.

We can study LZS interferometry!

Driven two level system



Schevchenko et. al. Phys. Rep. (2010)

$$\boxed{H = \frac{1}{2}\begin{pmatrix} \epsilon(t) & \Delta \\ \Delta & -\epsilon(t) \end{pmatrix}}$$

$$\boxed{\epsilon(t) = \beta t}$$

(Landau-Zener)

$$P_{LZ} = \exp[-\pi\Delta^2/2\beta]$$

$$\boxed{\epsilon(t) = A\sin(\omega t + \phi)}$$

(Landau-Zener-Stüeckelberg)

Driving around an anticrossing

Near the level crossing we can find an effective TLS:

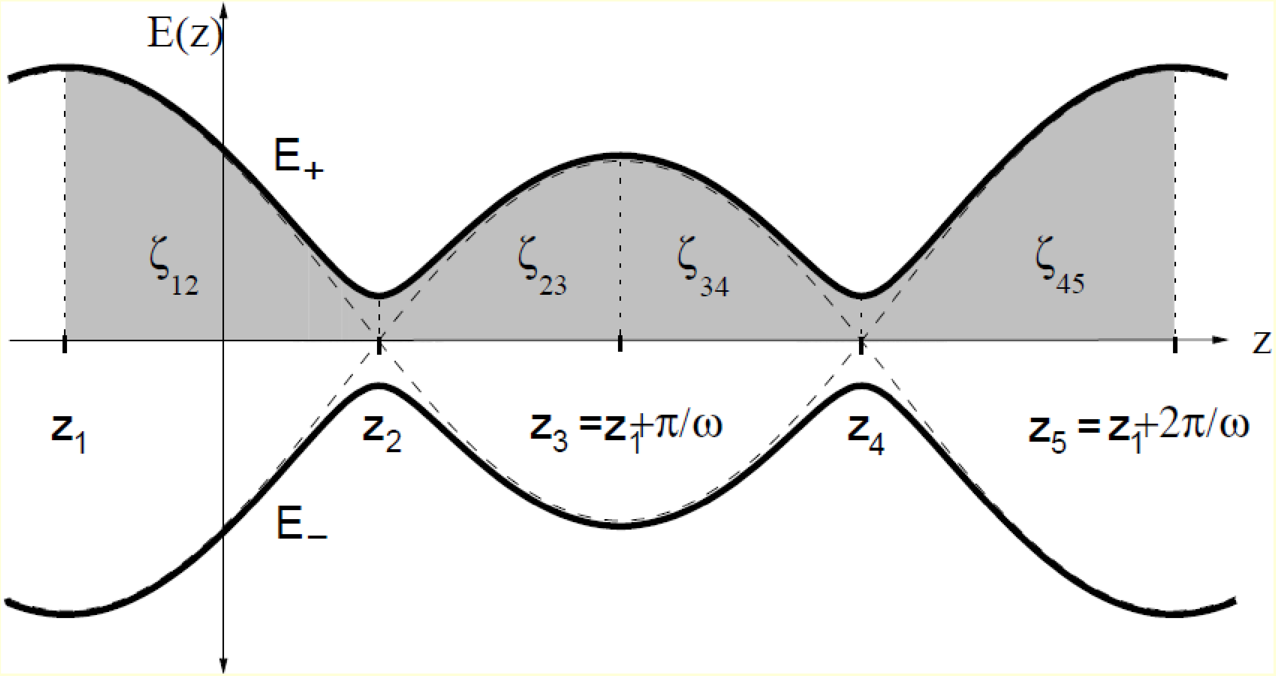
$$\boxed{H = \frac{1}{2}\begin{pmatrix} \epsilon & \Delta + \delta \\ \Delta - \delta & -\epsilon \end{pmatrix}}$$

(It is non-symmetric!)

$$\Delta = 2V_1 \qquad \delta = 2V_2$$

$$E_{\pm} = \pm\frac{1}{2}\sqrt{\epsilon^2 + (\Delta^2 - \delta^2)}$$

$$\epsilon = \epsilon_0 - \tilde{A}\sin(\tilde{\omega} z)$$



Non-symmetric LZS

An analytical aproximation can be obtained using the adiabatic-impulse model.

$$\mathbf{b}(t_a) = e^{-i\sigma_z \zeta(t_a)} N e^{-i\sigma_z \zeta(t_a)} \mathbf{b}(-t_a)$$

$$N = \begin{pmatrix} \sqrt{\frac{\Delta+\delta}{\Delta-\delta}}\sqrt{1-P_{LZ}}e^{-i\varphi_s} & \sqrt{P_{LZ}} \\ -\sqrt{P_{LZ}} & \sqrt{\frac{\Delta-\delta}{\Delta+\delta}}\sqrt{1-P_{LZ}}e^{i\varphi_s} \end{pmatrix}$$

Non-symmetric LZS

The system is prepared in the ground state and then driven several times through the avoided level crossing. We obtain the following analytical results:

$$\boxed{P_+^{even}(n) = 4\left(\frac{\Delta-\delta}{\Delta+\delta}\right) P_{LZ}(1-P_{LZ})\sin^2(\Phi_{St})\frac{\sin^2(n\phi)}{\sin^2(\phi)}}$$

$$\boxed{P_+^{odd}(n) = 2Q_1\frac{\sin^2(n\phi)}{\sin^2(\phi)} - Q_2\frac{\sin(2n\phi)}{\sin(\phi)} + P_{LZ}\cos(2n\phi)}$$

$$P_{LZ} \;=\; e^{-2\pi\gamma} \qquad \gamma = (\Delta^2-\delta^2)/4v$$

SR, L. Morales-Molina and F. Olivares, to appear in J. Phys. A.

Non-symmetric LZS

Critical limits:

$$\delta \rightarrow \Delta$$

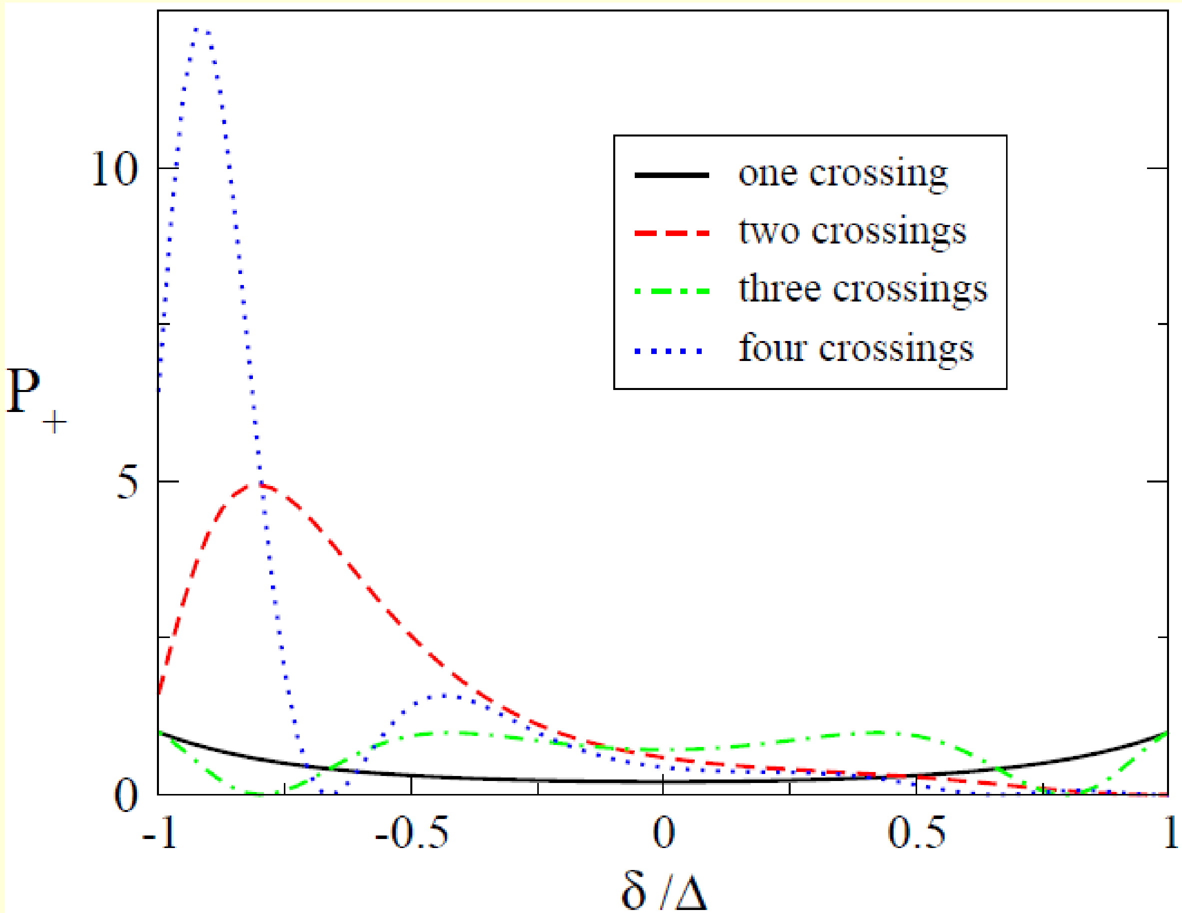
$$\boxed{\begin{aligned} P_+^{even}(n) &= 0 \\ P_+^{odd}(n) &= 1 \end{aligned}}$$

$$\delta \rightarrow -\Delta$$

$$\boxed{\begin{aligned} P_+^{even}(n) &= \frac{8\pi\Delta^2}{v}\sin^2(\Phi_{St})\frac{\sin^2(n(\zeta_{12}+\zeta_{45}-\zeta_{24}))}{\sin^2(\zeta_{12}+\zeta_{45}-\zeta_{24})} \\ P_+^{odd}(n) &= 1. \end{aligned}}$$

Non-symmetric LZS

$$\epsilon_0 = 0,\ \tilde{A} = 2,\ \tilde{\omega} = 0.02 \text{ and } \Delta = 2V_1 = 0.2$$

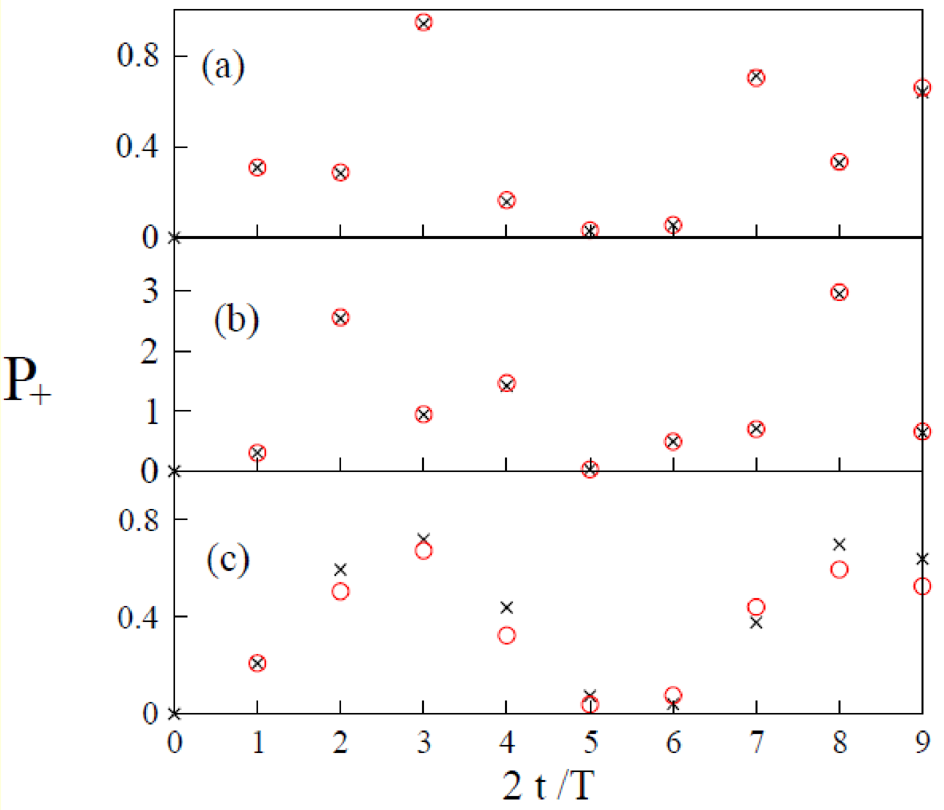


SR, L. Morales-Molina and F. Olivares, to appear in J. Phys. A.

Non-symmetric LZS

Results compare very well with numerical simulations of the full system.

$$\epsilon_0 = 0,\ \tilde{A} = 2,\ \tilde{\omega} = 0.02 \text{ and } \Delta = 2V_1 = 0.2$$



$$\delta = 2V_2 = 0.1$$

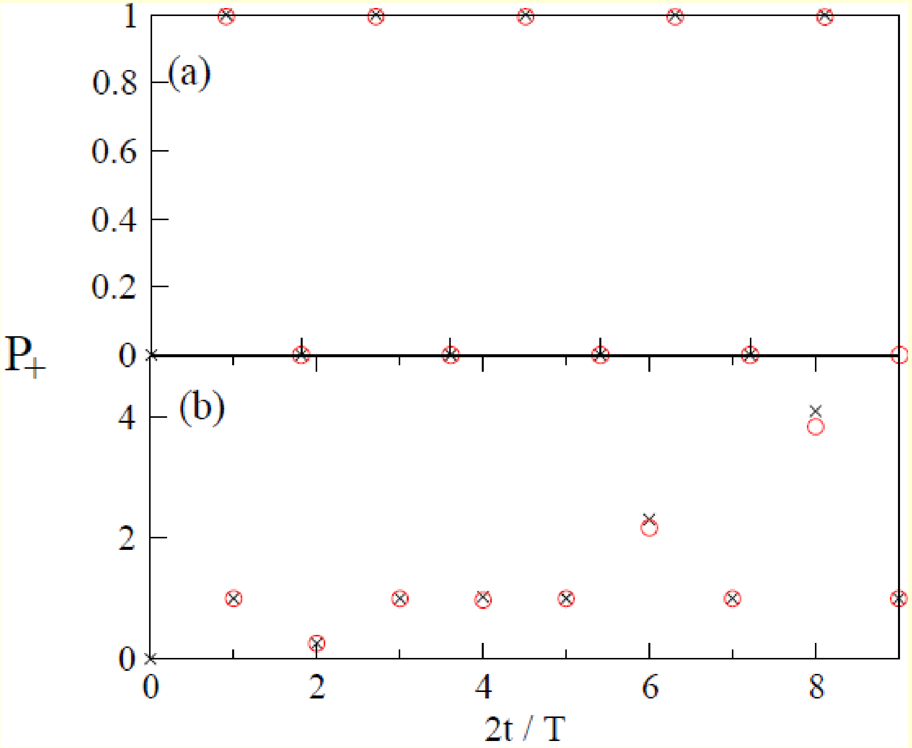
$$\delta = 2V_2 = -0.1$$

$$\delta = 2V_2 = 0$$

Non-symmetric LZS

Results compare very well with numerical simulations of the full system.

$$\epsilon_0 = 0,\ \tilde{A} = 3,\ \tilde{\omega} = 0.9 \text{ and } \Delta = 2V_1 = 0.2$$



$$\delta = 2V_2 = 1.999$$

$$\delta = 2V_2 = -1.999$$

Summary

- Engineered photonic waveguides can be used as classical analogues of *PT*-symmetric quantum mechanics.
- We study LZS transitions on a PT-symmetric optical lattice.
- Analytical calculations are possible for a non-symmetric two level system.
- The results reveal that the imaginary part of the potential δ, opens new possibilities for the control of the resulting mode energy.
- Can be used also for discrete systems…

Thank you!

$$\psi_q(x, Z) = \sum_l a_q^l(Z)\, \mathrm{e}^{\mathrm{i}(2kl+q)x}$$

$$\mathrm{i}\frac{\partial a_q^l}{\partial z} = (2l + \tilde{q})^2 a_q^l + (V_1 + V_2)a_q^{l+1} + (V_1 - V_2)a_q^{l-1}$$

$$\cos\phi = (1 - P_{LZ})\cos(\zeta_{15} + 2\varphi_S) + P_{LZ}\cos(\zeta_{12} + \zeta_{45} - \zeta_{24})$$

$$Q_1 = P_{LZ}\left[P_{LZ}\sin^2(\zeta_{12} + \zeta_{45} - \zeta_{24}) + (1 - P_{LZ})(1 - \cos(\zeta_{15} + 2\phi_S)\cos(\zeta_{12} + \zeta_{45} - \zeta_{24}))\right]$$

$$Q_2 = 2P_{LZ}(1 - P_{LZ})\sin(\varphi_S + \zeta_{12} + \zeta_{45})\sin(\varphi_S + \zeta_{24})$$