

PHHQP XI, Paris, 2012



# **Delay Times and Particlelike Scattering States in Systems with Loss and Gain**

Philipp Ambichl

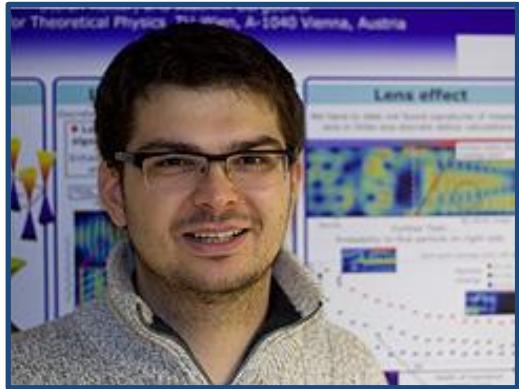
# Collaborators



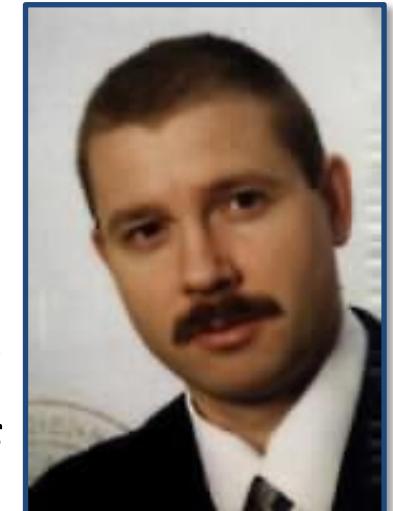
**Stefan Rotter,**  
TU Vienna



**Kostas Makris,**  
Princeton University



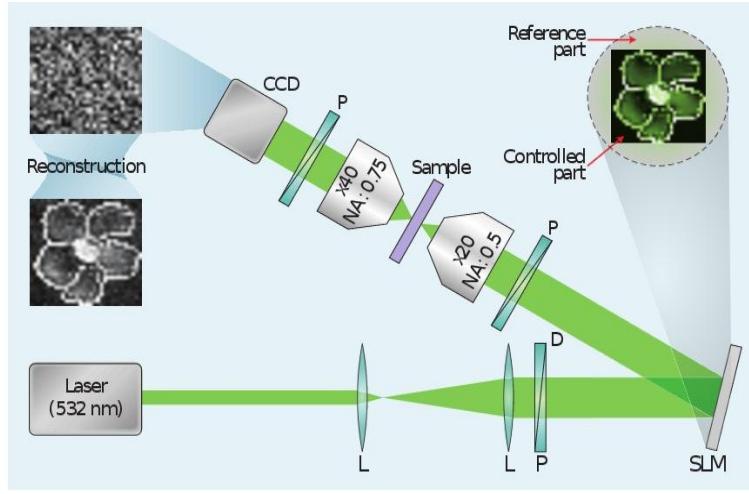
**Florian Libisch,**  
Princeton University



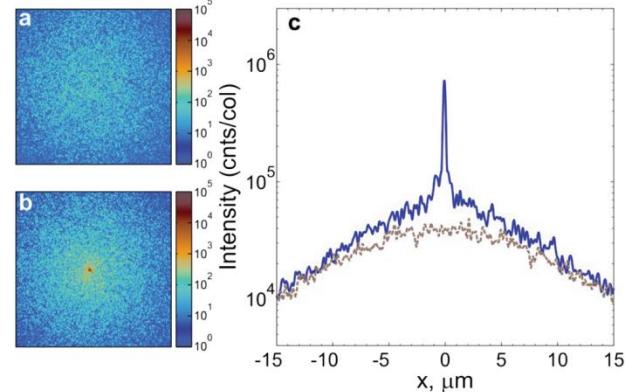
**Uwe Günther,**  
Helmholtz Center  
Dresden-Rosendorf

# Coherent Wave Shaping

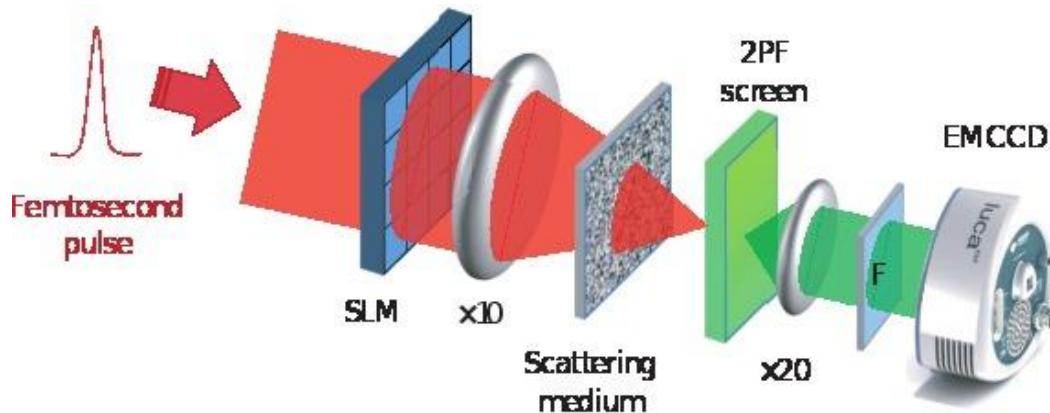
## image transmission<sup>1</sup>



## intensity enhancement<sup>2</sup>

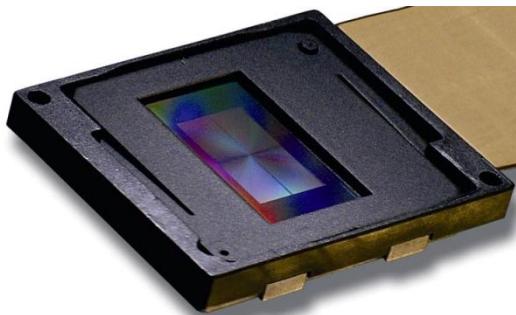


## spatial + temporal focussing<sup>3</sup>



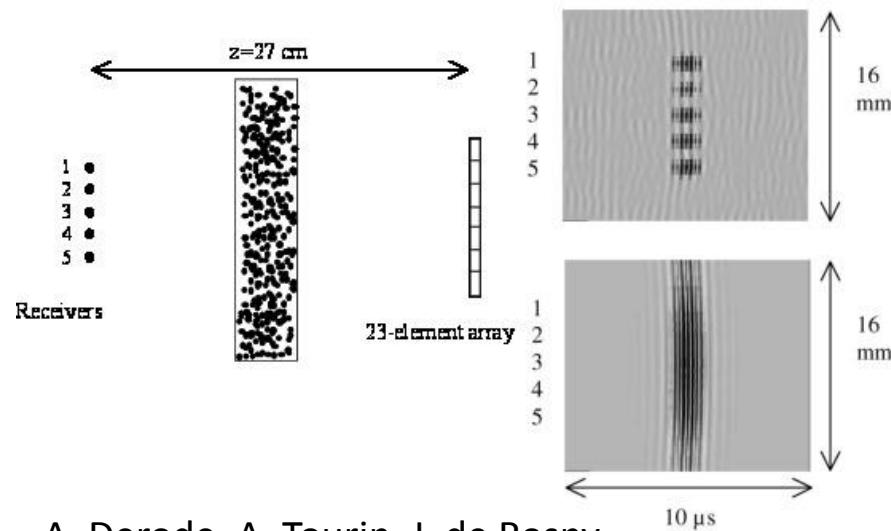
- [1] S.M. Popoff, G. Lerosey, M. Fink, Boccara, S. Gigain, Nature Commun. **1**, 1-5 (2010)
- [2] I.M. Vellekoop, A.P. Mosk, Phys. Rev. Lett. **101**, 120601 (2008)
- [3] O. Katz, E. Small, Y. Bromberg, Y. Silberberg, Nature Photonics **5**, 372 (2011)

# Coherent Wave Shaping



light waves

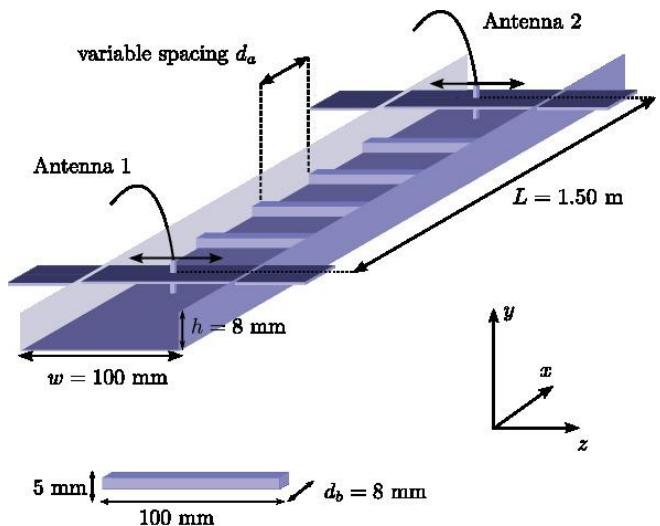
spatial light modulators (SLM)



A. Derode, A. Tourin, J. de Rosny,  
M. Tanter, S. Yon, M. Fink,  
Phys. Rev. Lett. **90**, 014301-1 (2003)

ultrasound

microwaves



O. Dietz, U. Kuhl, H.-J. Stöckmann,  
N.M. Makarov, F.M. Izrailev,  
Phys. Rev. B **83**, 134203 (2011)

# Time Delay

2-port system:

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}_{2N \times 2N}$$

scattering matrix

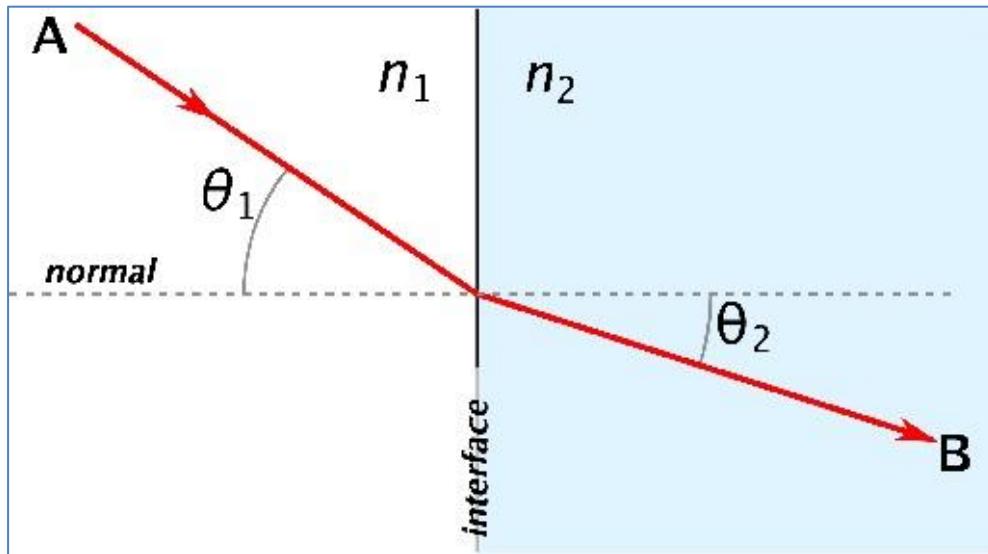
$$Q_{WS} = i\hbar \frac{dS^\dagger}{dE} S$$

Wigner-Smith  
time delay operator

**Leonard Eisenbud, Eugene Wigner, Felix Smith:**  
extract information about duration of scattering event from phase derivative

- L. Eisenbud, Dissertation, Princeton (1948)  
E.P. Wigner, Phys. Rev. **98**, 145 (1955)  
F.T. Smith, Phys. Rev. **118**, 349 (1960)
- P.W. Brouwer, K.M. Frahm, C.W.J. Beenakker, Phys. Rev. Lett **78**, 4737 (1997)
- V.V. Sokolov, V. Zelevinsky, Phys. Rev. C **56**, 311 (1997)
- S. Souma, A. Suzuki, Phys. Rev. B **65**, 115307 (2002)
- D.V. Savin, H.-J. Sommers, Phys. Rev. E **68**, 036211 (2003)
- M. Schultze et al., Science **328** 5986 (2010)
- Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, D.N. Christodoulides, Phys. Rev. Lett. **106**, 213901 (2011)

# Fermat's Principle



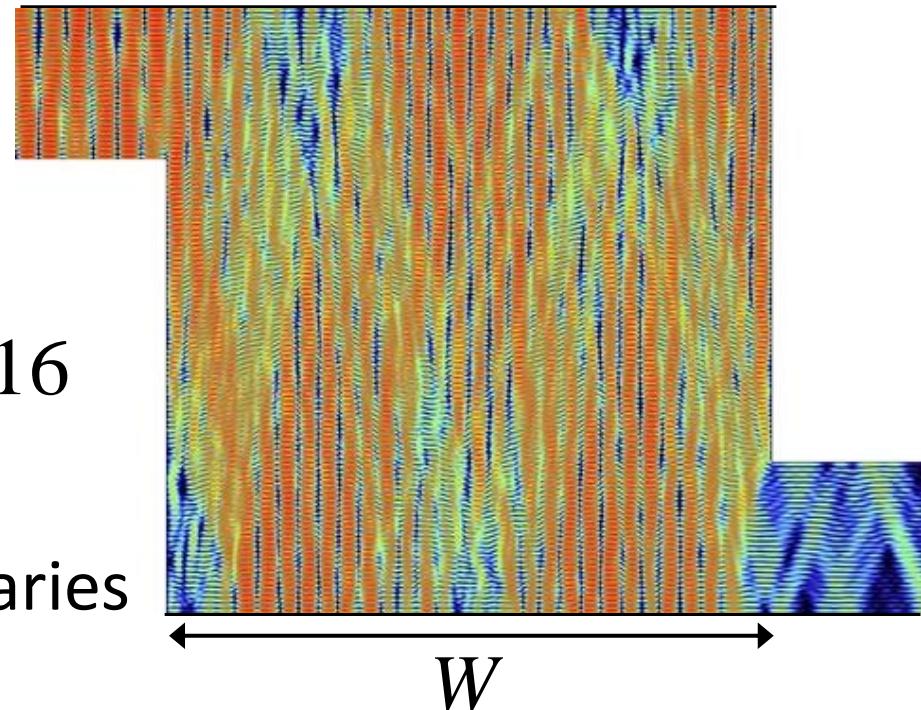
principle of  
least time



Pierre de Fermat

# Investigated System

wave injected  
through left  
wave-guide



$$\frac{\lambda}{W} \approx 0.016$$

hard-wall boundaries

- derivations in framework of Schrödinger equation
- applicable to Helmholtz equation, acoustic wave equation, etc.

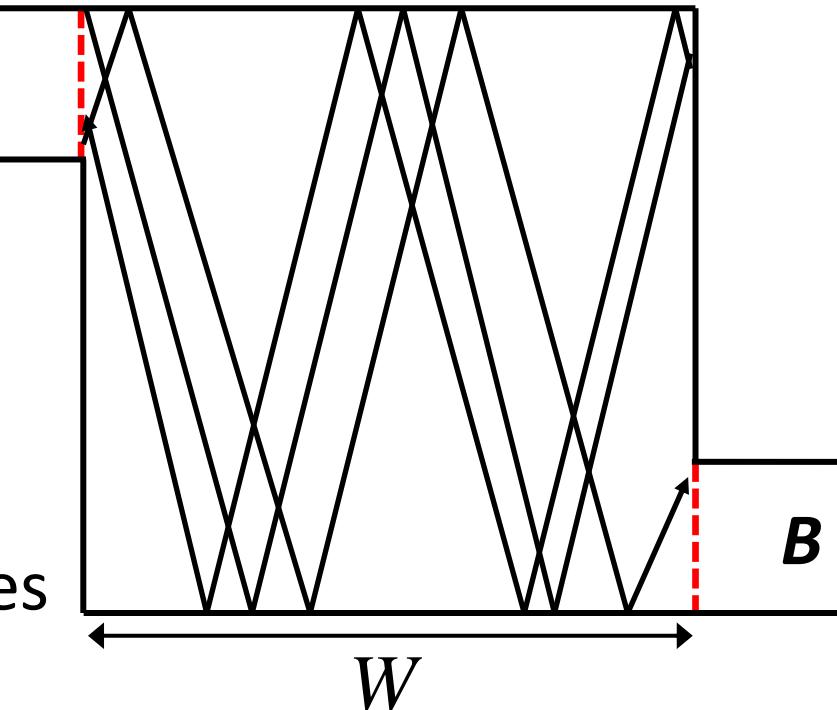
# Investigated System

wave injected  
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hard-wall boundaries



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# Algorithm

eigenvectors of  
time delay  
operator

$$Q_{WS} = i\hbar \frac{dS^\dagger}{dE} S := \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$$

eigenvectors  $Q_{WS}\vec{q} = \tau\vec{q}$

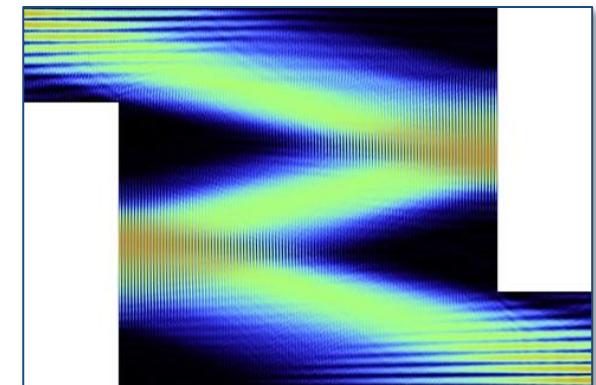
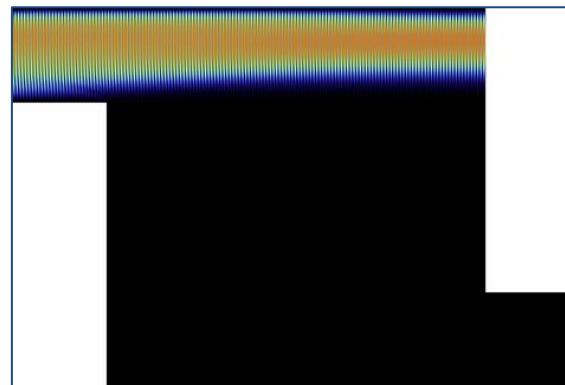
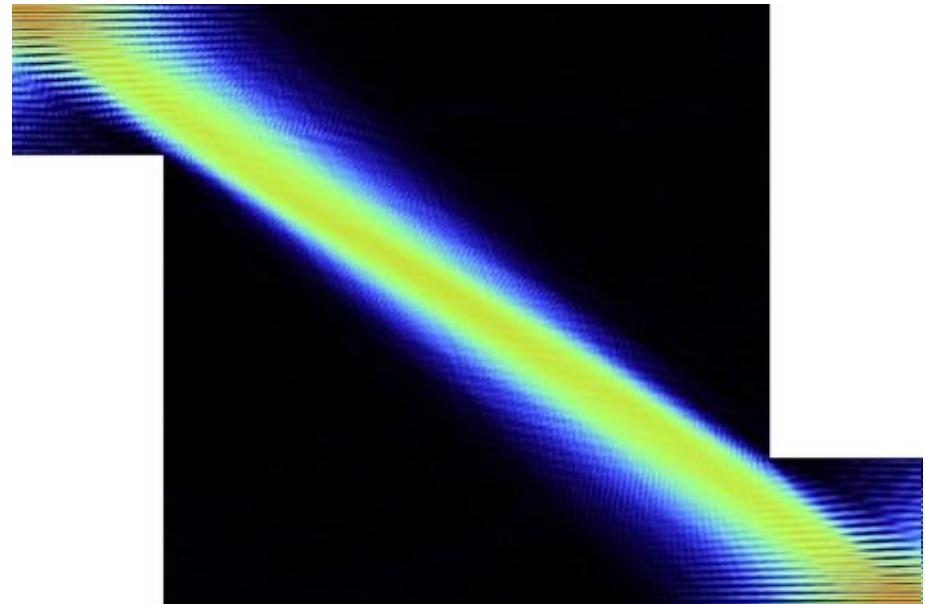
Injection only from left:  $\vec{q} = \begin{pmatrix} \vec{v} \\ 0 \end{pmatrix}$

$$Q_{WS}\vec{q} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} \begin{pmatrix} \vec{v} \\ 0 \end{pmatrix} = \boxed{\begin{pmatrix} Q_{11}\vec{v} \\ Q_{21}\vec{v} \end{pmatrix}} = \begin{pmatrix} \tau\vec{v} \\ 0 \end{pmatrix} = \tau\vec{q}$$

# Particlelike States

$$Q_{WS} = i\hbar \frac{dS^\dagger}{dE} S$$

time delay operator



S. Rotter, P. A., F. Libisch,  
Phys. Rev. Lett. **106**, 120602 (2011)

**Physical Review**  
**Focus**

# Particlelike States

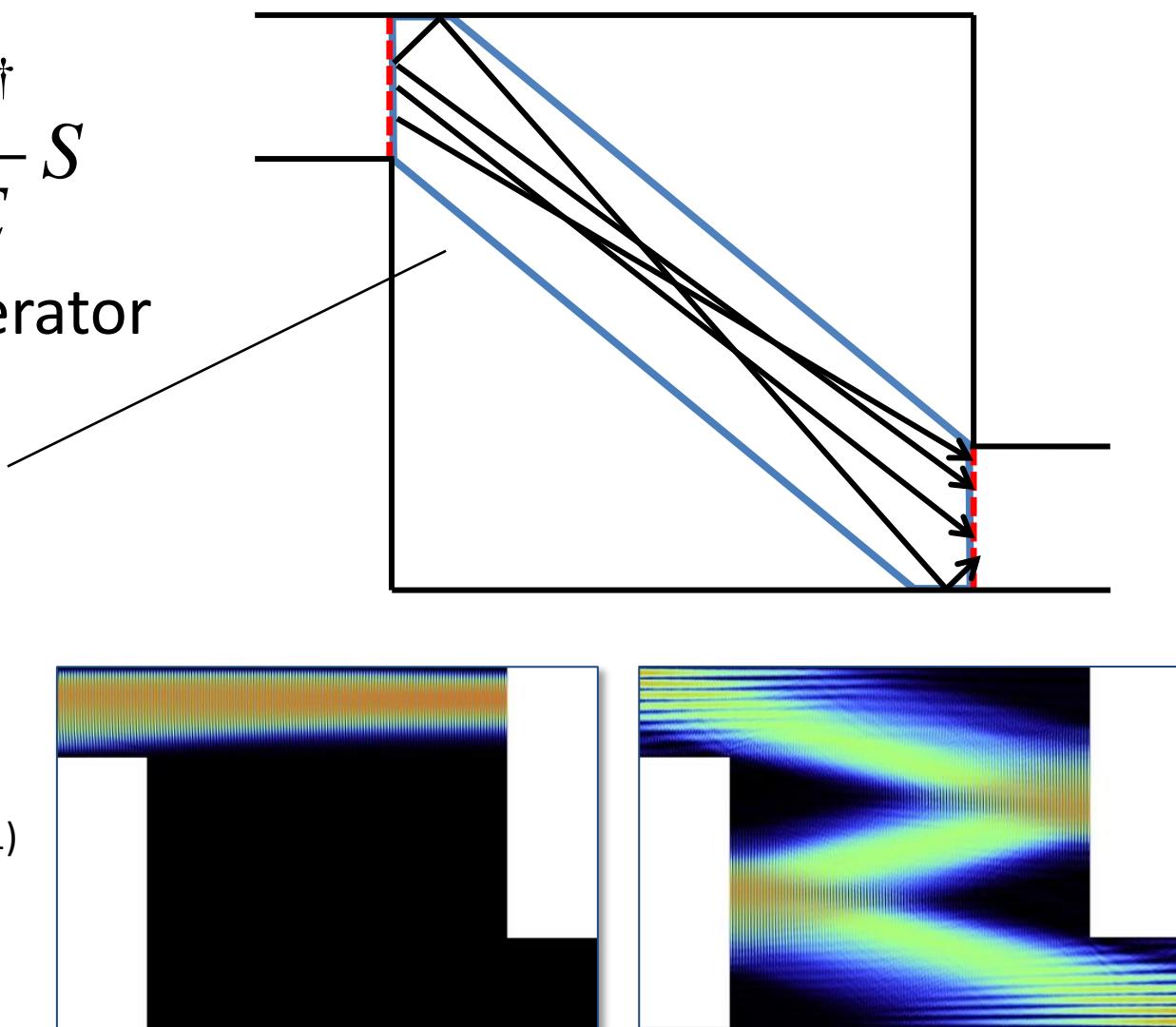
$$Q_{WS} = i\hbar \frac{dS^\dagger}{dE} S$$

time delay operator

bundles of  
particle-  
trajectories

S. Rotter, P. A., F. Libisch,  
Phys. Rev. Lett. **106**, 120602 (2011)

**Physical Review**  
**Focus**



# Time Delay

$$S^\dagger S \neq 1 \Rightarrow Q_{WS} \neq Q_{WS}^\dagger \quad \langle v | Q_{WS} | v \rangle \notin \Re$$

$$S = -1 + i\hbar V^\dagger G V$$

propagation                          coupling

flux not conserved?

$$G = \frac{1}{E - H_{eff}}$$

$$Q_{WS} = i\hbar \frac{dS^\dagger}{dE} S \approx \hbar^2 V^\dagger G^\dagger G V$$

V.V. Sokolov, V. Zelevinsky, Phys. Rev. C **56**, pp. 311-323 (1997)

# Dwell Time Operator

$$Q_d = \hbar^2 V^\dagger G^\dagger G V = (i\hbar G V)^\dagger (i\hbar G V)$$

dwell time  $\tau_d = \frac{\int |\psi(\vec{x})|^2 dV}{j_{in}} = \int |\psi(\vec{x})|^2 dV$

$$|\psi\rangle = i\hbar G V |v\rangle$$



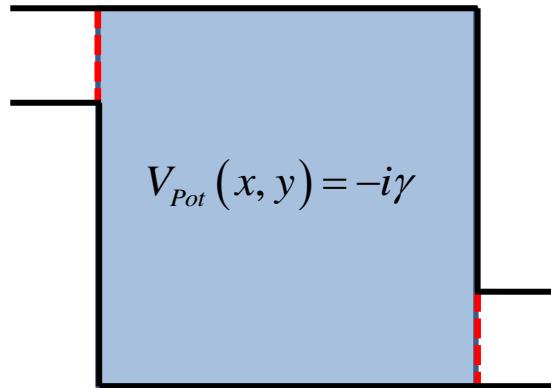
$$\tau_d = \int \psi^* \psi \, dV = \langle \psi | \psi \rangle = \langle v | Q_d | v \rangle$$

hermitian per  
construction!

physically  
meaningful!

# Uniform Loss

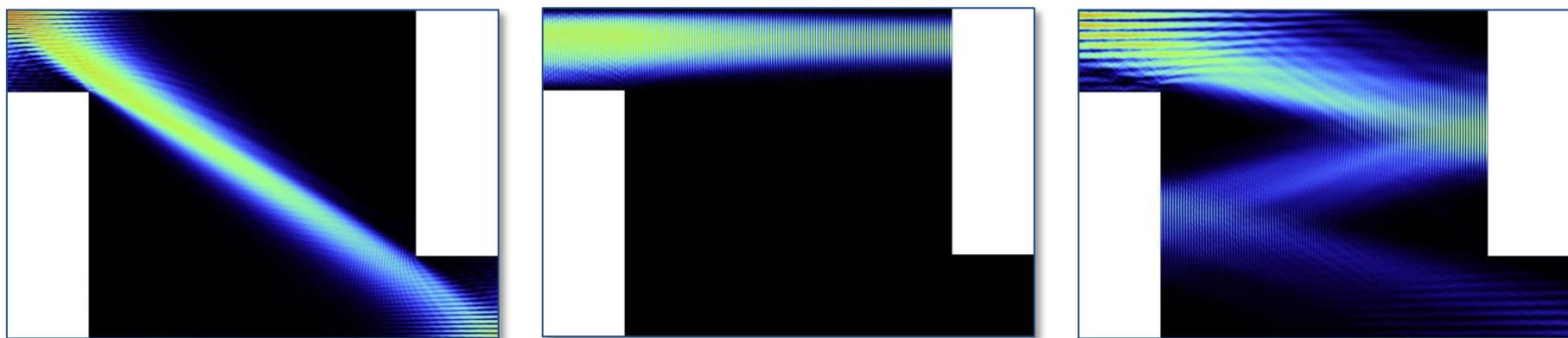
$$Q_d = \hbar^2 V^\dagger G^\dagger G V \rightarrow Q_d = \frac{1}{2\gamma} (1 - S^\dagger S)$$



absorption

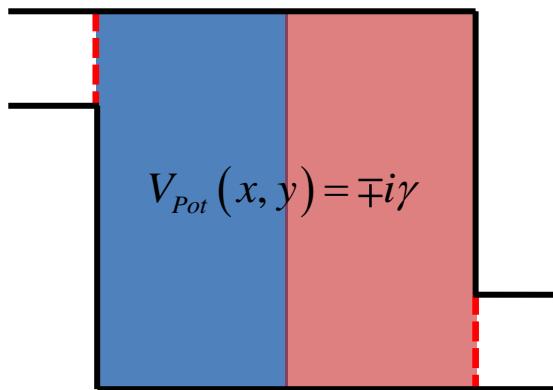
sub-unitarity

D.V. Savin, H.-J. Sommers, Phys. Rev. E **68**, 036211 (2003)



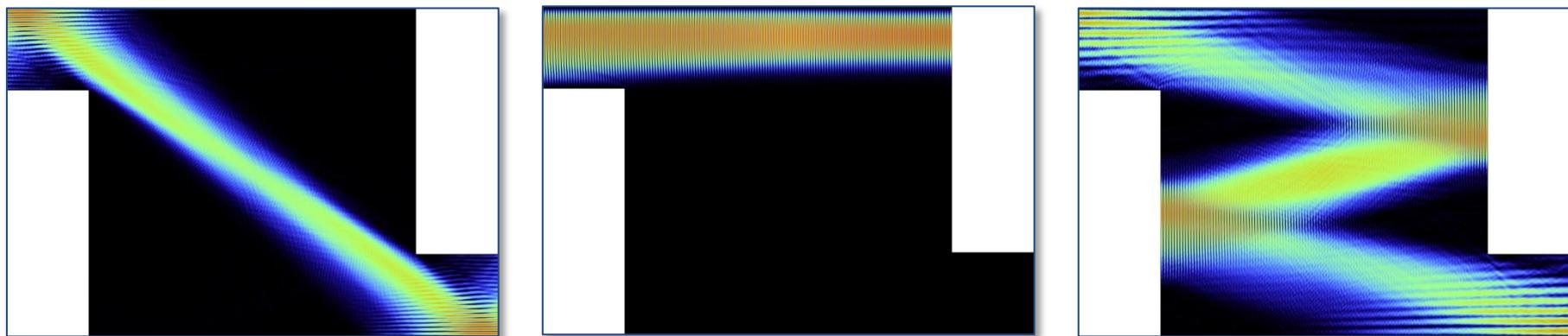
# PT Symmetry

$$Q_d = \hbar^2 V^\dagger G^\dagger G V$$



methods not  
applicable

alternative operator?



# PT Symmetry

Y.D. Chong, Li Ge, A.D. Stone, Phys. Rev. Lett. **106**, 093902 (2011)

$$S^\dagger S = 1$$

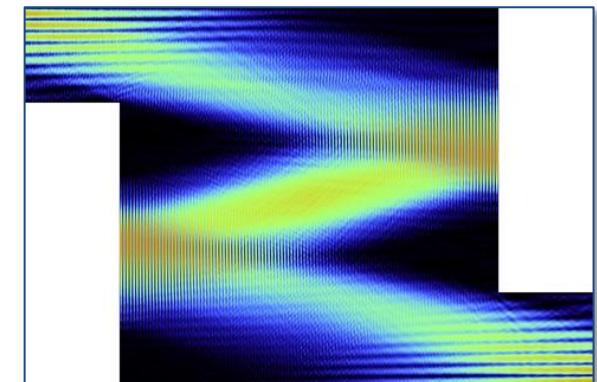
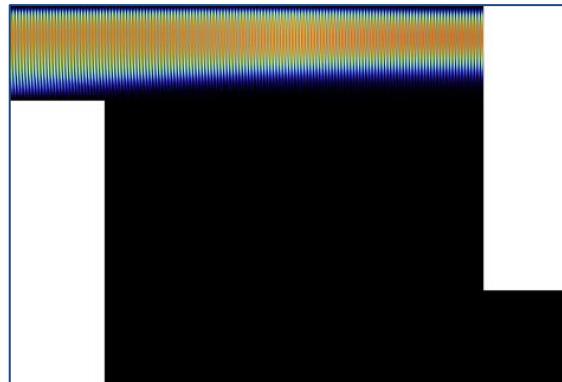
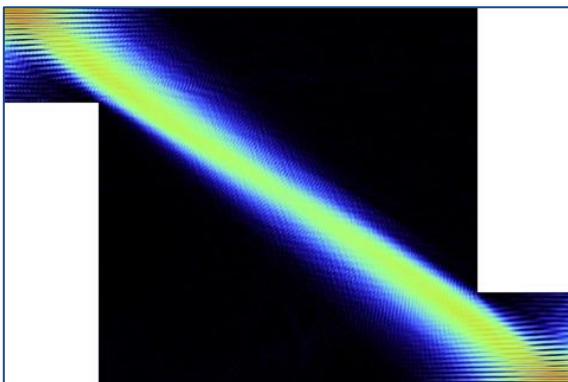
$$PS^\dagger PS = 1$$

$$Q_{WS} = i\hbar \frac{dS^\dagger}{dE} S$$

$$Q_{PT} = i\hbar P \frac{dS^\dagger}{dE} PS$$

$$Q_{WS} = Q_{WS}^\dagger$$

$$Q_{PT} = P Q_{PT}^\dagger P$$



# Summary

- particlelike scattering states in unitary systems:  
**eigenstates of  $Q_{WS}$**
- **highly collimated beams**
- approximation for  $Q_{WS}$  is **dwell time operator**
- **hermitian** per construction
- particlelike scattering states in **uniformly absorptive** and **PT-symmetric** systems
- **generalization of  $Q_{WS}$**  on PT-symmetric systems