Effects of nonuniform strain on PT-symmetric photonic crystals

Selectively breaking PT symmetry for topologically protected states

Henning Schomerus
Lancaster University
PHHQP Paris, 29 August 2012

Essence of this talk

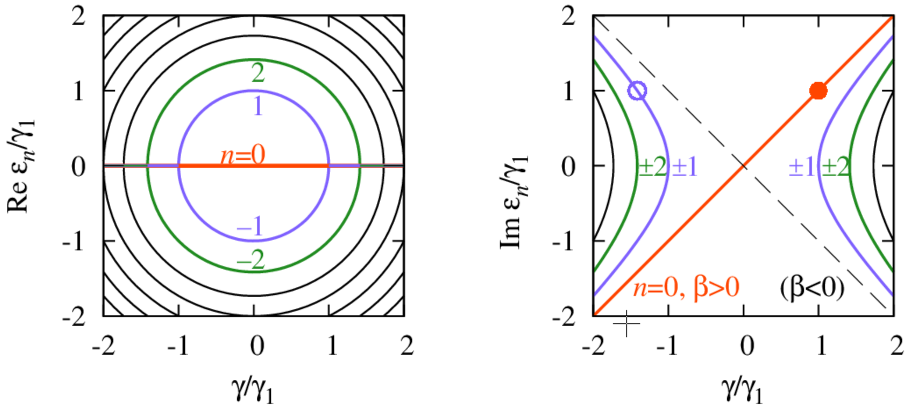
Consider the following Hamiltonian $\mathcal{H} = \begin{pmatrix} i\gamma & a\Pi^\dagger \\ a\Pi & -i\gamma \end{pmatrix}$ where $[\Pi, \Pi^\dagger] = 1$.

This delivers a spectrum of 'Landau levels'

0^{th} level $\phi_0 = \begin{pmatrix} \chi_0 \\ 0 \end{pmatrix}$, $\varepsilon_0 = i\gamma$, where $\Pi\chi_0 = 0$

higher Landau levels for $n = \pm1, \pm2, \pm3, \ldots$ $\phi_n = \begin{pmatrix} \chi_{|n|} \\ \alpha_n \chi_{|n|-1} \end{pmatrix}$, $\varepsilon_n = \mathrm{sgn}(n)\sqrt{\gamma_n^2 - \gamma^2}$,

$$\alpha_n = \mathrm{sgn}(n)\sqrt{1 - \frac{\gamma^2}{\gamma_n^2}} - i\frac{\gamma}{\gamma_n}, \quad \gamma_n = \sqrt{a^2|n|},$$



The 0^{th} level is anamolous,

all others are paired in $\mathcal{PT}$-symmetric fashion.

The Hamiltonian above is *not* $\mathcal{PT}$-symmetric.

- Realized in a *locally* $\mathcal{PT}$-symmetric photonic lattice.
- Is then a manifestation of the *parity anomaly*.
- Can be used to produce an interesting *laser* or *filter*.

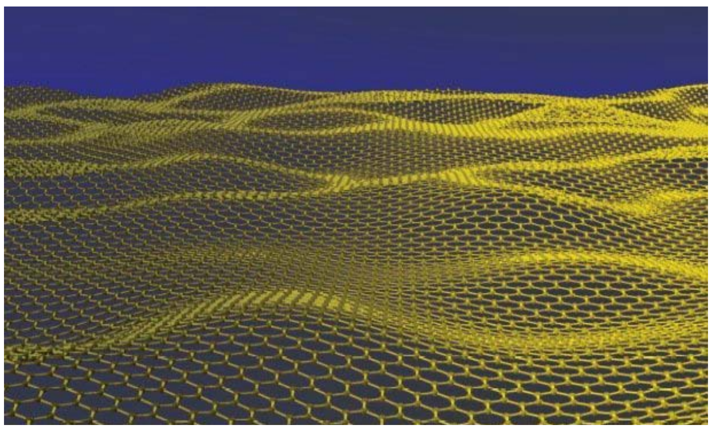
Dirac electrons from honeycomb lattices: graphene

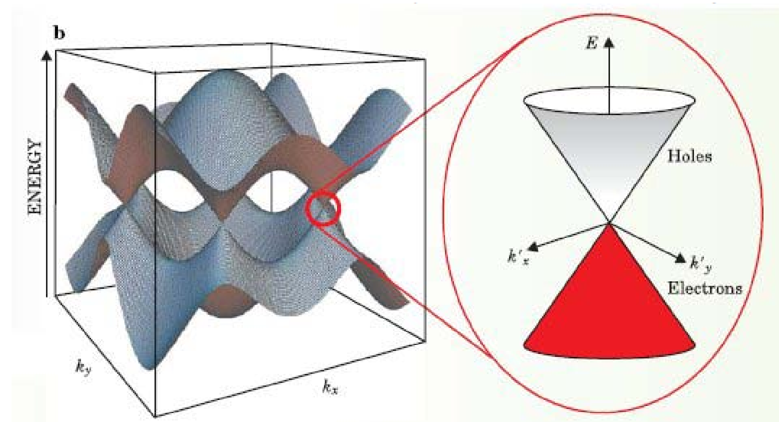
Vol 438|10 November 2005|doi:10.1038/nature04233 nature

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]

lattice | dispersion | Dirac cones $\varepsilon = \pm v|\mathbf{q}|$





$\varepsilon = \pm v|\mathbf{q}|$, $\mathbf{q} = \mathbf{k} - \mathbf{K}_\sigma$ near K points $\mathbf{K}_\sigma = \sigma(4\pi/3\sqrt{3}\rho)\mathbf{i}$,

where $\sigma = \pm 1$ distinguishes two independent *valleys*

whose existence is guaranteed by the $\mathcal{P}$ (inversion) and $\mathcal{T}$ symmetries

Dirac electrons from honeycomb lattices: graphene, or photonic sys.

PRL 98, 103901 (2007) PHYSICAL REVIEW LETTERS week ending 9 MARCH 2007

Conical Diffraction and Gap Solitons in Honeycomb Photonic Lattices

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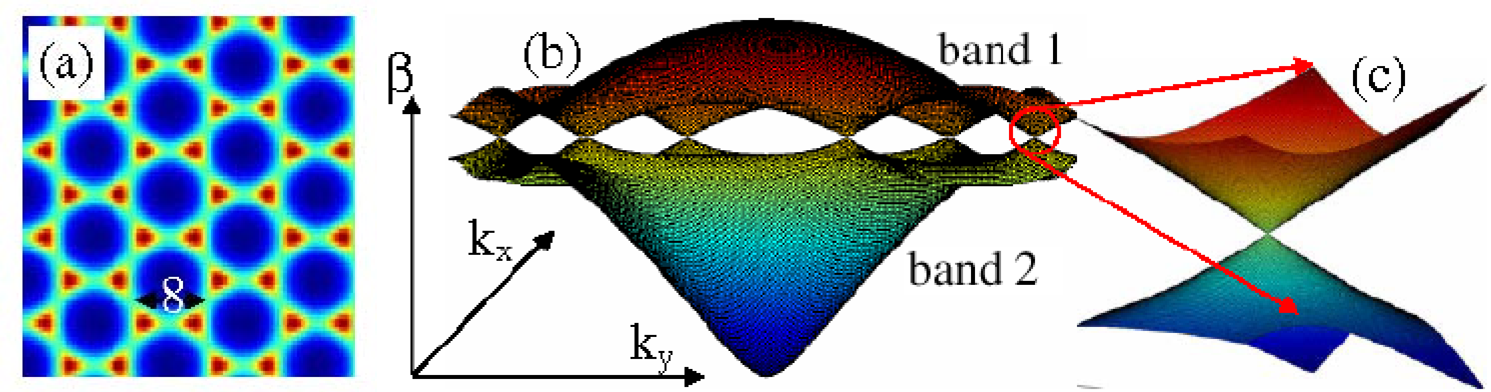

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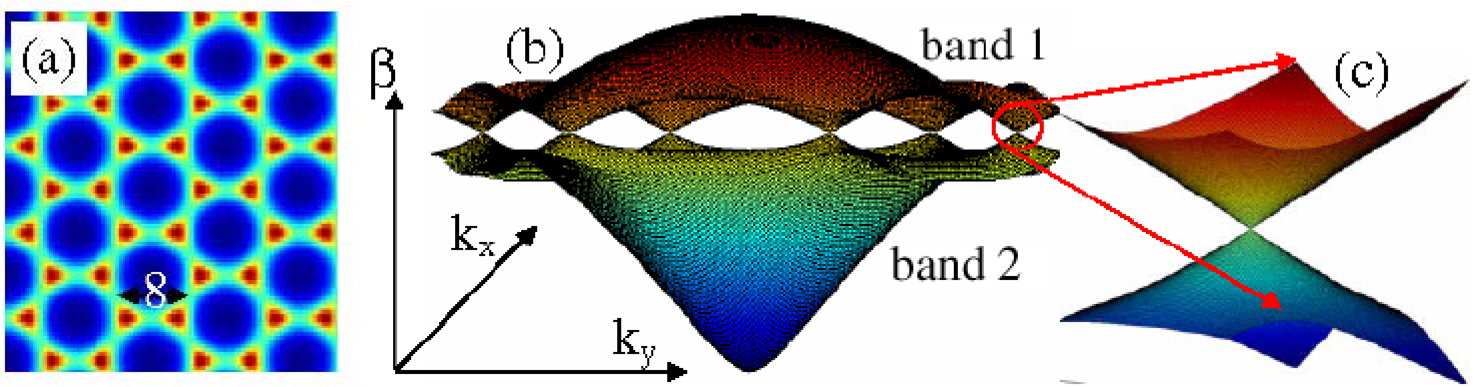


tight binding model $\Rightarrow$ Dirac equation

$$H = \sum_{<ab>} t_{ab}(|a\rangle\langle b| + |b\rangle\langle a|) \qquad \mathcal{H} = v\begin{pmatrix} 0 & \sigma p_x - ip_y \\ \sigma p_x + ip_y & 0 \end{pmatrix}$$

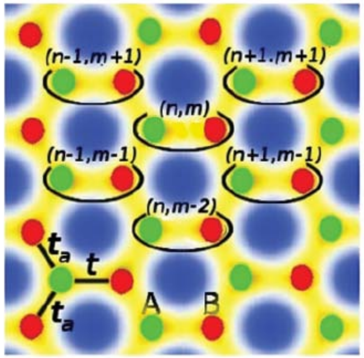
Break hermiticity

$$\mathcal{H} = \begin{pmatrix} i\gamma & \sigma v(p_x - ip_y) \\ \sigma v(p_x + ip_y) & -i\gamma \end{pmatrix}$$

Still PT symmetric

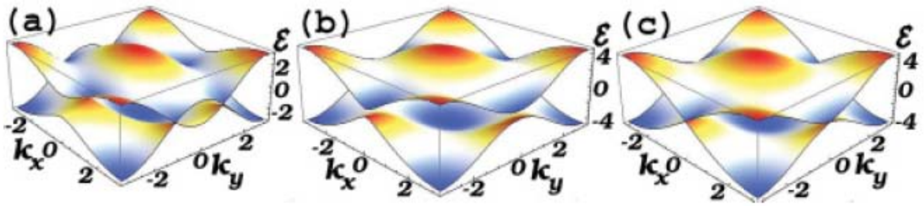
$$\mathcal{H}(x,y) = \sigma_x \mathcal{H}^*(-x,-y)\sigma_x \equiv \mathcal{PT} H(x,y) \mathcal{PT}$$

PHYSICAL REVIEW A 85, 013818 (2012)



Exceptional-point dynamics in photonic honeycomb lattices with $\mathcal{PT}$ symmetry

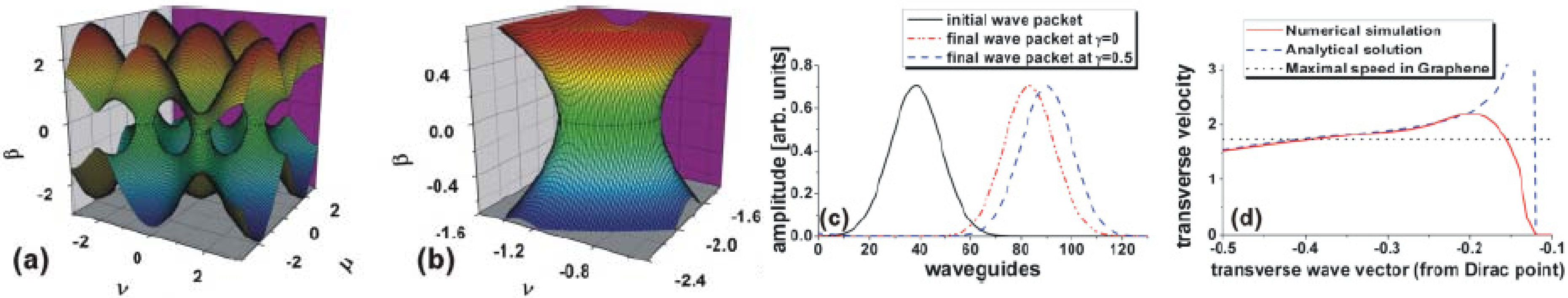
Hamidreza Ramezani,[1] Tsampikos Kottos,[1,2] Vassilios Kovanis,[3] and Demetrios N. Christodoulides[4]



PHYSICAL REVIEW A 84, 021806(R) (2011)

$\mathcal{PT}$-symmetry in honeycomb photonic lattices

Alexander Szameit, Mikael C. Rechtsman, Omri Bahat-Treidel, and Mordechai Segev



Landau levels: Quantum Hall effect in graphene

Dirac electron in a magnetic field ($B = B_z > 0$, $\sigma = 1$):

$$\mathcal{H} = c \begin{pmatrix} 0 & P_x - iP_y \\ P_x + iP_y & 0 \end{pmatrix} \quad \text{where } \mathbf{P} = -i\nabla - \mathbf{A}, \quad \mathbf{A} = (B/2)(-y\mathbf{i} + x\mathbf{j}) \Rightarrow \boxed{\mathcal{H} = c\sqrt{2B} \begin{pmatrix} 0 & \Pi^\dagger \\ \Pi & 0 \end{pmatrix}}$$

with $[\Pi, \Pi^\dagger] = 1$

$$\text{0th Landau level } \phi_0 = \begin{pmatrix} \chi_0 \\ 0 \end{pmatrix}, \quad \varepsilon_0 = 0, \text{ where } \Pi\chi_0 = 0$$

$$\text{Higher LL } \phi_n = \begin{pmatrix} \chi_{|n|} \\ \mathrm{sgn}(n)\chi_{|n|-1} \end{pmatrix}, \quad \varepsilon_n = \mathrm{sgn}(n)\sqrt{2c^2B|n|}, \quad n = \pm1, \pm2, \pm3, \ldots$$

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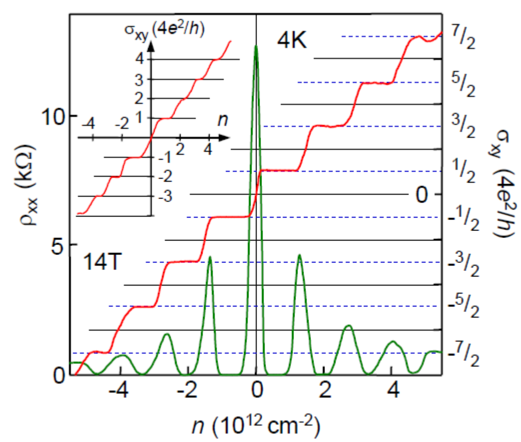

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Higher LL $\phi_n = \begin{pmatrix} \chi_{|n|} \\ \mathrm{sgn}(n)\chi_{|n|-1} \end{pmatrix}$, $\quad \varepsilon_n = \mathrm{sgn}(n)\sqrt{2c^2B|n|}$, $\quad n = \pm 1, \pm 2, \pm 3, \ldots$

Parity anomaly (Jackiw, Semenoff, Haldane)

when $\sigma \to -\sigma$
or $B \to -B$:

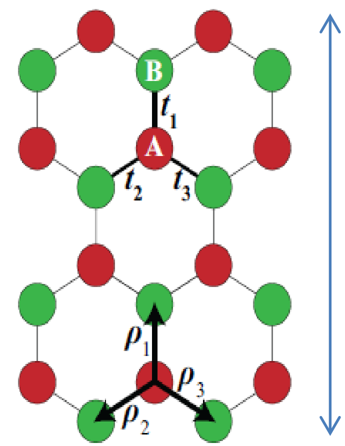
$$\mathcal{H} = c\sqrt{2|B|}\begin{pmatrix} 0 & \Pi \\ \Pi^\dagger & 0 \end{pmatrix} \quad \phi_0 = \begin{pmatrix} 0 \\ \chi_0 \end{pmatrix}, \quad \varepsilon_0 = 0$$

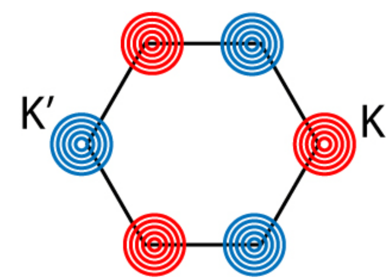
Combine QHE and PT symmetry

Problems: cannot use magnetic fields

- Parity anomaly compensated in both valleys
- Want gain and loss, thus photons, but magnetic effects on those are small

Solution: *Pseudomagnetic fields via strain*





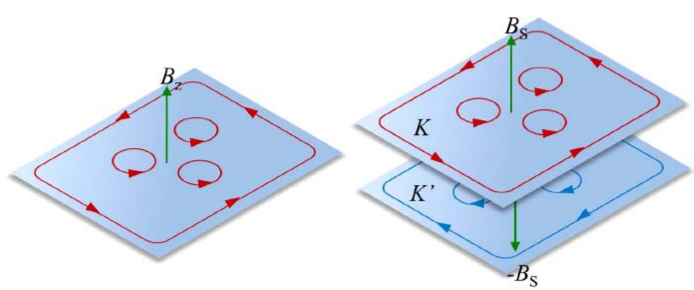
Uniform strain displaces Dirac points from K points:

Shift in momentum, equivalent to vector potential

$$\mathbf{A} = \sigma \frac{1}{3\rho t_0}(2t_1 - t_2 - t_3)\mathbf{i} + \sigma \frac{1}{\sqrt{3}\rho t_0}(t_2 - t_3)\mathbf{j}$$

opposite in both valleys (valley-Hall effect)

nonuniform strain creates effective magnetic field



$$t_l = t_0[1 - (\beta/2)\boldsymbol{\rho}_l \cdot \mathbf{r}], \quad l = 1, 2, 3,$$

gives rise to $\mathbf{A} = (\sigma\beta/2)(-y\mathbf{i} + x\mathbf{j})$.

LETTERS

PUBLISHED ONLINE: 27 SEPTEMBER 2009 | DOI:10.1038/NPHYS1420

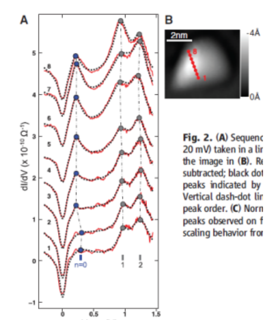
nature physics

Energy gaps and a zero-field quantum Hall effect in graphene by strain engineering

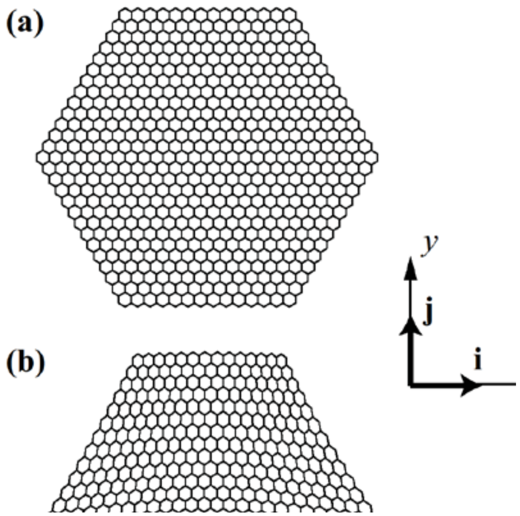
F. Guinea[1]*, M. I. Katsnelson[2] and A. K. Geim[3]*

Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy,[1,2]*† S. A. Burke,[1]*‡ K. L. Meaker,[1] M. Panlasigui,[1] A. Zettl,[1,2] F. Guinea,[3] A. H. Castro Neto,[4] M. F. Crommie[1,2]§



Model



sublattices A and B have amplification rates γ_A and γ_B,

$$\begin{aligned}\gamma_A &= \bar{\gamma} + \gamma \\ \gamma_B &= \bar{\gamma} - \gamma\end{aligned}$$

$\bar{\gamma}$ is the average rate

γ quantifies the imbalance

$$H = i\gamma_A \sum_a |a\rangle\langle a| + i\gamma_B \sum_b |b\rangle\langle b| + \sum_{<ab>} t_{ab}(|a\rangle\langle b| + |b\rangle\langle a|).$$

Uniform strain displaces Dirac points from K points:

Shift in momentum, equivalent to vector potential

$$\mathbf{A} = \sigma\frac{1}{3\rho t_0}(2t_1 - t_2 - t_3)\mathbf{i} + \sigma\frac{1}{\sqrt{3}\rho t_0}(t_2 - t_3)\mathbf{j}$$

$$\mathcal{H} = \begin{pmatrix} i\gamma_A & v(\sigma P_x - iP_y) \\ v(\sigma P_x + iP_y) & i\gamma_B \end{pmatrix}$$

where $\mathbf{P} = -i\nabla - \mathbf{A}$

$t_l = t_0[1 - (\beta/2)\boldsymbol{\rho}_l \cdot \mathbf{r}], \quad l = 1, 2, 3,$
gives rise to $\mathbf{A} = (\sigma\beta/2)(-y\mathbf{i} + x\mathbf{j})$.

$$\mathcal{H} = \begin{pmatrix} i\gamma_A & v\sqrt{2\beta}\Pi^\dagger \\ v\sqrt{2\beta}\Pi & i\gamma_B \end{pmatrix}, \quad \begin{matrix} \beta > 0 \\ [\Pi, \Pi^\dagger] = 1 \end{matrix}$$

$$\Pi = \frac{1}{\sqrt{2\beta}}(-i\sigma\beta x/2 - i\sigma\partial_x + \beta y/2 + \partial_y)$$

Model

$$\mathcal{H} = \begin{pmatrix} i\bar{\gamma} + i\gamma & v\sqrt{2\beta}\Pi^\dagger \\ v\sqrt{2\beta}\Pi & i\bar{\gamma} - i\gamma \end{pmatrix}, \quad \Pi = \frac{1}{\sqrt{2\beta}}(-i\sigma\beta x/2 - i\sigma\partial_x + \beta y/2 + \partial_y), \quad [\Pi, \Pi^\dagger] = 1$$

Solution

$$\chi_0 = (\beta/2\pi)^{1/2}\exp[-\beta(x^2+y^2)/4 + \lambda(\sigma x + iy) - \lambda^2/\beta]$$

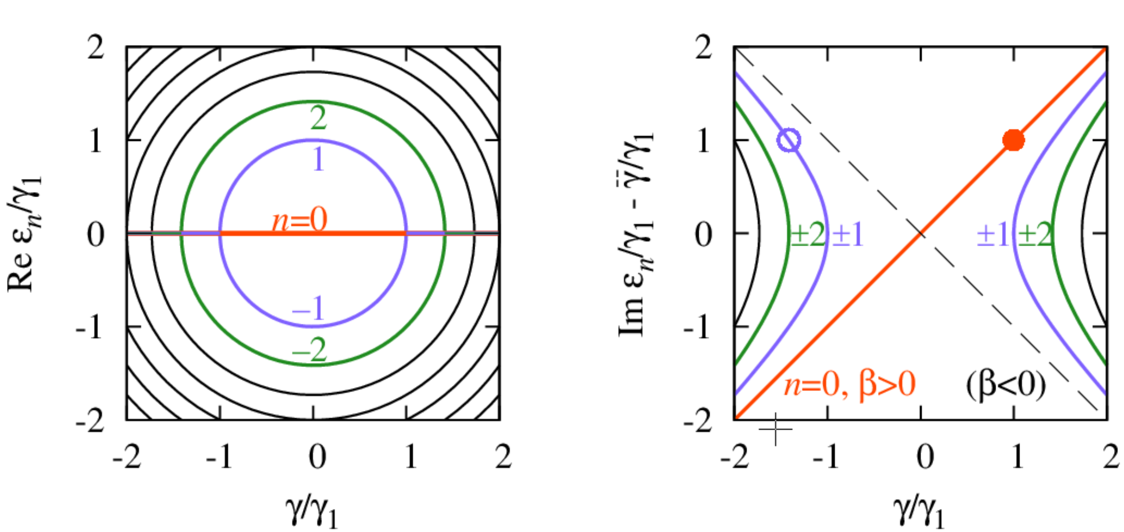
$$\Pi\chi_0 = 0 \qquad \chi_m = (m!)^{-1/2}(\Pi^\dagger)^m\chi_0$$

$$\phi_0 = \begin{pmatrix} \chi_0 \\ 0 \end{pmatrix}, \quad \varepsilon_0 = i\gamma_A = i\bar{\gamma} + i\gamma,$$

$$\phi_n = \begin{pmatrix} \chi_{|n|} \\ \alpha_n\chi_{|n|-1} \end{pmatrix}, \quad \varepsilon_n = i\bar{\gamma} + \mathrm{sgn}(n)\sqrt{\gamma_n^2 - \gamma^2},$$

$$\alpha_n = \mathrm{sgn}(n)\sqrt{1 - \frac{\gamma^2}{\gamma_n^2}} - i\frac{\gamma}{\gamma_n}$$

$$\gamma_n = \sqrt{2v^2\beta|n|}$$



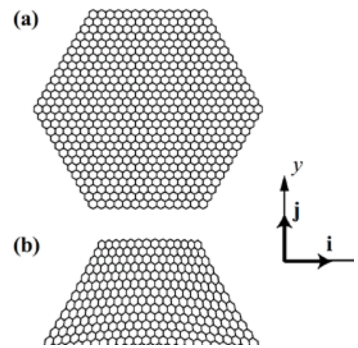
Explanation of broken (n=0) and emerging (|n|>0) PT symmetry?

$n = 0$: direct consequence of parity anomaly. Exploit SUSY (Jackiw):

$$(\varepsilon - i\gamma_A)\varphi_A = (\varepsilon - i\gamma_B)^{-1} 2\beta v^2 \Pi^\dagger \Pi \varphi_A,$$

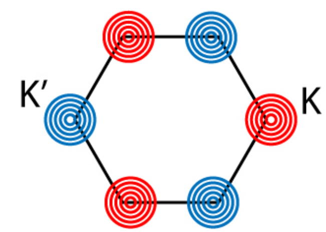
$$(\varepsilon - i\gamma_B)\varphi_B = (\varepsilon - i\gamma_A)^{-1} 2\beta v^2 \Pi \Pi^\dagger \varphi_B,$$

$$\Pi \Pi^\dagger = \Pi^\dagger \Pi + 1$$



reflection symmetry $x \to -x$, $\sigma \to -\sigma$

replicated identically in both valleys



$|n| > 0$: as if the system were $\mathcal{PT}$-symmetric

$\mathcal{H} = \mathcal{PTHPT} + 2i\bar{\gamma} \Rightarrow \mathrm{Im}\varepsilon_n = \bar{\gamma}$, or pairs ε_n, $\varepsilon_{-n} = \varepsilon_n - 2i\mathrm{Im}\,\varepsilon_n + 2i\bar{\gamma}$

however, does not apply to $n = 0$

construct generalized symmetry $\widetilde{\mathcal{PT}}$ in space of higher LL: antilinear, $\widetilde{\mathcal{PT}}^2 = 1$

Details of construction

introduce the basis $|m,A\rangle \equiv \begin{pmatrix} \chi_m \\ 0 \end{pmatrix}, \quad |m,B\rangle \equiv \begin{pmatrix} 0 \\ \chi_m \end{pmatrix}, \quad m = 0, 1, 2, \ldots$

$\mathcal{H} = i\gamma_A|0,A\rangle\langle 0,A| + \widetilde{\mathcal{H}}$, where

$$\widetilde{\mathcal{H}} = \sum_{m=0}^{\infty} \Big(i\gamma_A|m+1,A\rangle\langle m+1,A| + i\gamma_B|m,B\rangle\langle m,B|$$
$$+ v\sqrt{2\beta(m+1)}(|m+1,A\rangle\langle m,B| + |m,B\rangle\langle m+1,A|) \Big)$$

$$\widetilde{\mathcal{P}} = \sum_{m=0}^{\infty}(|m+1,A\rangle\langle m,B| + |m,B\rangle\langle m+1,A|)$$

$$\widetilde{\mathcal{T}} : \Gamma|m,L\rangle \to \Gamma^*|m,L\rangle, \quad L = A, B,$$

$\widetilde{\mathcal{T}}^2 = 1, \ \widetilde{\mathcal{P}}^\dagger = \widetilde{\mathcal{P}}, \ \widetilde{\mathcal{P}}^2 = 1 - |0,A\rangle\langle 0,A|$ take $\widetilde{\mathcal{PT}} \equiv \widetilde{\mathcal{P}}\widetilde{\mathcal{T}}|_{n\neq 0}$

$\boxed{\widetilde{\mathcal{H}} \doteq \widetilde{\mathcal{PT}}\widetilde{\mathcal{H}}\widetilde{\mathcal{PT}} + 2i\bar{\gamma}}$ $\quad \widetilde{\mathcal{P}}\widetilde{\mathcal{T}}\mathcal{H}\widetilde{\mathcal{P}}\widetilde{\mathcal{T}} = \mathcal{H} - 2i\bar{\gamma} - 2i\gamma|0,A\rangle\langle 0,A|$

Possible signatures:

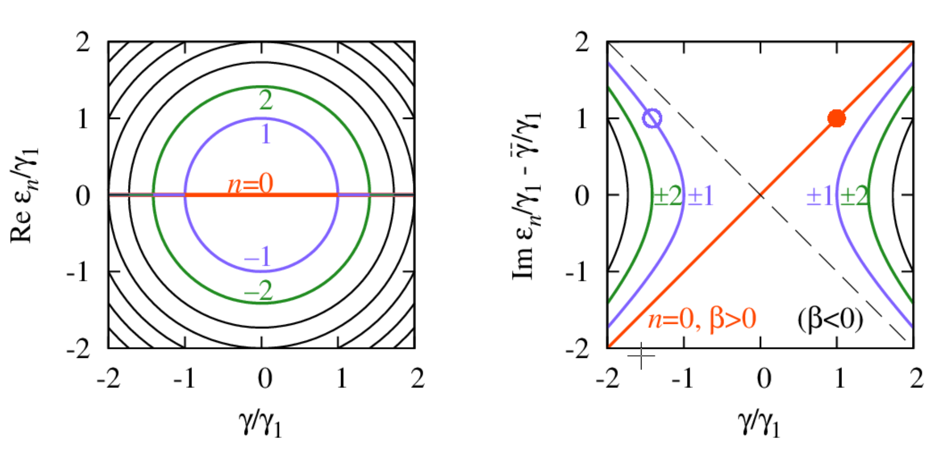
- Lasers: fix $\bar{\gamma} < 0$; for $\gamma = 0$ system is passive (uniformly absorbing)

as $|\gamma|$ increases: $\mathrm{Im}\,\varepsilon_0$ changes, $\mathrm{Im}\,\varepsilon_n$ fixed until γ_n.

Laser threshold depends on sign of $\gamma\beta$.

$\gamma\beta > 0$: 0^{th} level on amplifying lattice, threshold at $|\gamma| = |\bar{\gamma}|$.

$\gamma\beta < 0$: need to wait for amplification of 1st level, threshold at $|\gamma| = \sqrt{2v^2|\beta| + \bar{\gamma}^2}$.



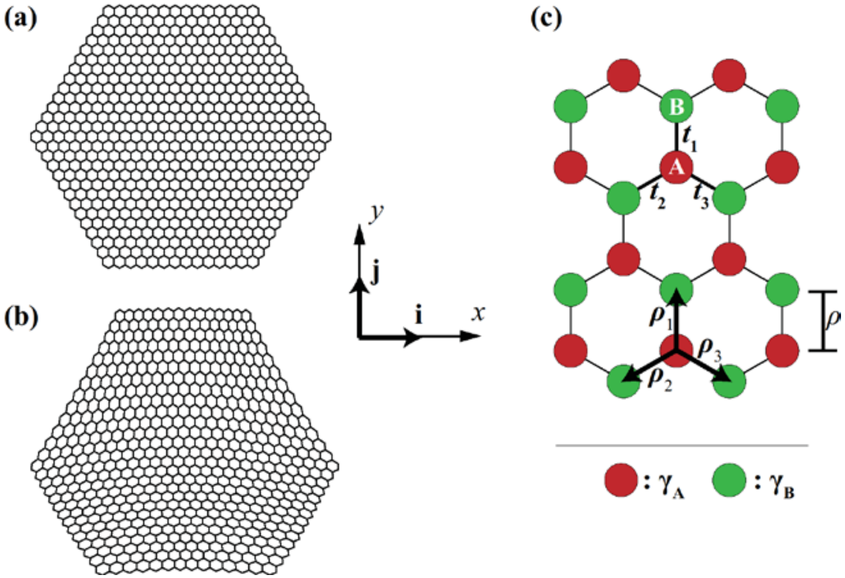
lasing level different in both cases

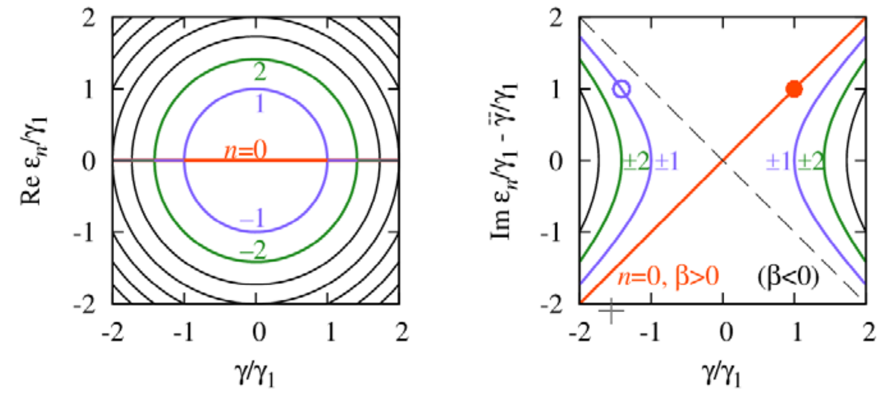
modes still degenerate $\rightarrow$ lifted by mode competition (finite system)

- filters: beam dynamics in 3D setting

for $0 < |\gamma| < \gamma_1$: 0^{th} level selectively amplified or damped

Summary: Locally PT-symmetric photonic system





realizes parity anomaly and an emerging (dynamical?) PT symmetry

gives a selectively amplified (infinitely degenerate) state

arxiv:1208.2901, with Nicole Yunger Halpern

other aspects of parity vs inversion symmetry in 2D PT-sym systems:
1207.1454 (scattering theory), 1208.2575 (random matrix theory)